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THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

VOLUME IV

**GENERAL INSTABILITY OF CYLINDERS HAVING
LONGITUDINAL AND CIRCUMFERENTIAL
STIFFENERS; AXIAL COMPRESSION**

Prepared for the
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ABSTRACT

This is the fourth of six volumes, each bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The complete set of documents was prepared under NASA Contract NAS8-11181. This particular volume deals with the general instability of simply supported, axially compressed circular cylinders which have longitudinal and circumferential stiffening (stringers and rings). Analysis methods are presented in the forms of curves and a digital computer program. Since the contents of this volume are based upon a Donnell-type small-deflection theory, the proposed methods should be used in conjunction with empirical knock-down factors to account for the effects of initial imperfections. In addition, the Donnell assumptions preclude application to non-axisymmetric buckle patterns where the number of circumferential waves is small.

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DEFINITION OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
A_r	Cross-sectional area of single ring (no cylindrical skin included); (see Table III and its notes).
A_{r+s}	Cross-sectional area of single ring augmented by an effective width of skin which is considered to essentially behave as part of the ring (see Table III and its notes).
A_s	Cross-sectional area of single stringer (no cylindrical skin included); (see Table III and its notes).
$A_{11}, A_{22}, A_{12}, A_{33}$	Elastic constants (see Table III).
a	Ring spacing.
B_{A12}	See equations (2-14) and inequality (2-15).
b	Stringer spacing.
b_s	Thickness of integral longitudinal stiffener (see Table III).
C_R	Correction factor defined by equation (3-5).
C_F	Experimentally determined constant [see equation (1-1)].
C_{hx}	Quantity defined by equation (2-25).

DEFINITION OF SYMBOLS
(Continued)

<u>Symbol</u>	<u>Definition</u>
C_{hy}	Quantity defined by equation (2-29).
$C_{\rho x}$	Quantity defined by equation (2-26).
$C_{\rho y}$	Quantity defined by equation (2-30).
$C_{11}, C_{22}, C_{12}, C_{21}$	Eccentricity coupling constants (see Table III).
c	Thickness of sandwich core.
D	Diameter of middle surface of basic cylindrical skin in conventional skin-stringer-ring constructions; Parameter defined in equations (A-1).
D_Q	Transverse shear stiffness of sandwich wall [see equations (A-1)].
$D_{11}, D_{22}, D_{12}, D_{33}$	Elastic constants (see Table III).
E	Young's modulus.
E_F	Young's modulus of sandwich faces.
G	Modulus of elasticity in shear.
G_c	Shear modulus of sandwich core.
h	Distance between middle surfaces of faces in sandwich cylinder.
h_s	Depth of integral longitudinal stiffener (see Table III).
h_x	See Figure 2.
h_y	See Figure 3.

DEFINITION OF SYMBOLS
(Continued)

<u>Symbol</u>	<u>Definition</u>
I	Moment of inertia.
\bar{I}_r	Centroidal moment of inertia for single ring (no cylindrical skin included); (see notes following Table III).
\bar{I}_{r+s}	Centroidal moment of inertia of single ring augmented by an effective width of skin which is considered to essentially behave as part of the ring (see Table III and its notes).
\bar{I}_x	Running centroidal moment of inertia of effective shell wall cross section lying in plane normal to axis of revolution (see Table III and its notes).
\bar{I}_y	Running centroidal moment of inertia of effective shell wall cross section lying in radial plane.
i	$= \sqrt{-1}$; symbol denoting the i^{th} quantity of an array of values.
K	Parameter defined by equation (3-7).
k_{x_a}	Sandwich loading parameter defined in equations (A-1).
L	Overall length of cylinder (see Figure 1).
M	Overall bending moment.
m	Number of axial half-waves in buckle pattern.

DEFINITION OF SYMBOLS
(Continued)

<u>Symbol</u>	<u>Definition</u>
N_s	Number of discrete load points for the case shown in Figure 4(a); (Number of stringers).
\bar{N}_{THIEL}	Loading parameter defined in equations (2-3), (positive for tensile loading).
$\left(\bar{N}_{THIEL}\right)_c$	$= - \bar{N}_{THIEL}$, (positive for compressive loading).
\bar{N}_x	Applied longitudinal tensile running load acting at the centroid of the effective skin-stringer combination.
$\left(\bar{N}_x\right)_c$	Applied longitudinal compressive running load acting at the centroid of the effective skin-stringer combination ($= - \bar{N}_x$).
n	Number of circumferential full-waves in buckle pattern.
R	Radius to middle surface of basic cylindrical skin in conventional skin-stringer-ring constructions; Radius to centroid of ring depicted in Figure 4; Mean radius of sandwich cylinder; Mean radius of isotropic cylinder.
r_a	Parameter defined in equations (A-1).
t	Thickness of basic cylindrical skin.
t_F	Thickness of single face of sandwich wall.
t_c	Skin thickness of corrugated wall.

DEFINITION OF SYMBOLS
(Continued)

<u>Symbol</u>	<u>Definition</u>
\bar{t}_x	Thickness of appropriate smeared-out area of cross section lying in plane normal to axis of revolution (see Table III and its notes).
\bar{t}_y	Thickness of appropriate smeared-out area of cross section lying in radial plane (see Table III and its notes).
U	Parameter defined by equation (2-10).
u,v,w,	Reference-surface displacements (see Figure 1).
W	Force depicted in Figure 4(a).
w_c	Running load depicted in Figure 4(b).
x,y,z	Coordinates (see Figure 1).
Z	Parameter defined by equation (2-2).
Z_a	Parameter defined in equations (A-1).
Z_l	Parameter defined by equation (2-7).
\bar{z}_x	Eccentricity (see Figure 2 and Table III).
\bar{z}_y	Eccentricity (see Figure 3 and Table III).
α	Parameter defined in equations (2-3).
β	Parameter defined in equations (2-3).
γ	Parameter defined in equations (2-3).
Δ_R	Radial deflection for the points of load application shown in Figure 4(a).

DEFINITION OF SYMBOLS
(Continued)

<u>Symbol</u>	<u>Definition</u>
δ_R	Radial deflection due to uniformly distributed running load shown in Figure 4(b).
η_p	Parameter defined in equations (2-3).
η_s	Parameter defined in equations (2-3).
θ	Half-angle between discrete load points depicted in Figure 4(a).
ν	Poisson's ratio.
ν_F	Poisson's ratio for faces of sandwich cylinder.
ρ_x	Local centroidal radius of gyration for effective shell wall cross section lying in a plane which is normal to the axis of revolution (see Table III and its notes).
ρ_y	Local centroidal radius of gyration for effective shell wall cross section lying in a radial plane (see Table III and its notes).
ρ_{xy}	Parameter defined in equations (2-13).
(Σd_i)	Total peripheral length of corrugation centerline.
σ_{cr}	Critical compressive buckling stress of isotropic cylinder.
$\bar{\sigma}$	Compressive stress obtained by dividing the longitudinal compressive running load by \bar{t}_x .

SECTION 1

INTRODUCTION

The contents of this volume deal with the general instability (see GLOSSARY, Volume I [1]) of simply supported, axially compressed circular cylinders having both stringers and rings. Designing for the prevention of this mode of instability usually centers around the choice of a suitable criterion to establish dimensions for the circumferential stiffeners. In the past, a number of rather crude empirical formulas have been proposed for this purpose. One of the earliest of these was proposed by the Guggenheim Aeronautical Laboratory of the California Institute of Technology (GALCIT) as an outgrowth of their tests on small-scale cylinders [2]. Shortly thereafter, the Polytechnic Institute of Brooklyn Aeronautical Laboratory (PIBAL) proposed a different criterion based on their own test results from similar specimens [3]. Shanley [4] then drew upon both the GALCIT and the PIBAL data to generate the following empirical formula for the minimum ring stiffness required to prevent general instability in stiffened cylinders subjected to pure bending:

$$EI_r = \frac{C_f MD^2}{a} \quad (1-1)$$

where

- E = Young's modulus
- \bar{I}_r = Centroidal moment of inertia for ring
- C_f = Experimentally determined constant
- M = Overall bending moment
- D = Cylinder diameter
- a = Ring Spacing

Numbers in brackets [] in the text denote references listed in SECTION 7.

Although derived specifically for the case of pure bending, this formula has been widely used for cases of axial compression by considering the peak running load intensity to be the controlling factor. Still another criterion was suggested by Becker [5] in 1958. This approach employs certain geometrical features of the inward-bulge along with an estimate of the elastic restraints afforded by the rings. In general, all of these approaches constitute oversimplifications of the problem in that they do not recognize all the important variables involved. Engineers have long been wary of these criteria and have hedged their ring designs through the use of generous safety factors and extensive proof-testing.

In recent years it has become increasingly popular to attack this problem by means of orthotropic shell theory. For example, applications have been made [6] of the small-deflection formulation presented by Thielemann in reference 7. The methods presented in the sections to follow are based on an extension of this approach to include the effects of finite cylinder length as well as stringer and ring eccentricities. These influences have also been included in an alternative formulation developed by Block, Card, and Mikulas in reference 8. Furthermore, Block [9] has recently published a paper which treats the buckling of eccentrically stiffened perfect orthotropic cylinders in which prebuckling deformations, load eccentricity, and ring discreteness are all considered. That paper points the way for further improvement of the methods presented in the sections to follow. It should be noted here that, under NASA Contract NAS 8-9500, the Lockheed Missiles and Space Company has likewise published a solution [10] which accounts for prebuckling deformations and load eccentricities (end moments). However, their development is limited to configurations which are stiffened only in the longitudinal direction (no intermediate rings).

The analysis methods presented in this volume are given in the forms of curves and a digital computer program. In order to keep the number of curves within reasonable bounds, they embody a number of simplifications which result in some loss of rigor. Therefore, these curves are to be used

primarily for the purposes of preliminary sizing, rough checking, and the study of trends. To meet the more stringent requirements of a final design, one should employ the digital computer program of SECTION 6. This program can be used to obtain single-point solutions and/or additional curves which more closely apply to the selected configurations of interest.

It should be kept in mind that this volume deals only with the general instability mode of failure. Other possible modes must be separately checked. The most important of these other possibilities is the panel instability (see GLOSSARY, Volume I [1]) mode which is treated in Volume III [11].

The theoretical foundation for the methods given here lies in small-deflection orthotropic shell theory. Therefore, it is recommended that these methods be used in conjunction with empirical knock-down factors in order to account for the influences of initial imperfections. Volume V [12] presents a practical criterion in this regard. Since most practical stiffened cylinders are "effectively thick", the related reductions will generally not be nearly so severe as those encountered for thin-walled isotropic cylinders.

It is also important to note that the basic buckling equation of this volume is based upon Donnell-type simplifications [13]. As a result, the methods given here cannot be applied when the instability manifests itself in a non-axisymmetric buckle pattern having a small number of circumferential waves. The rule-of-thumb guideline is offered here that these methods should be considered inapplicable for cases where

$$0 < n < 2 \quad (1-2)$$

Throughout this volume, only the case of classical simple support is considered. In addition, it is always assumed that the behavior is elastic. To consider critical stresses which lie above the proportional limit, one must employ iterative procedures which recognize the shape of the applicable stress-strain curve.

SECTION 2

EQUATIONS

2.1 GENERAL

The following orthotropic cylinder equation provides the basis for the methods given in this volume:

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p \sqrt{\gamma} \beta^2 + \gamma \beta^4\right]}{4\alpha \beta^4} + \frac{\alpha \beta^4 (z)^2}{\left[1 + 2\eta_s \beta^2 + \beta^4\right]} \quad (2-1)$$

where,

$$z = \left[1 - \frac{C_{11} + C_{22}}{2\alpha (A_{22} D_{22})^{1/2} \beta^2} - \frac{C_{12}}{2\alpha A_{22} (D_{22}/A_{11})^{1/2}} - \frac{C_{21}}{2\alpha (A_{11} D_{22})^{1/2} \beta^4} \right] \quad (2-2)$$

A detailed derivation of these relationships is given in reference 14 where the coordinate system shown in Figure 1 was used. Some general background information concerning equations (2-1) and (2-2) is given in Volume I [1]. As noted there, these equations have been written in rather compact, instructive forms through the introduction of the following parameters, most of which were first proposed by Thielemann [7]:

$$\left(\bar{N}_{THIEL}\right)_c = -\bar{N}_{THIEL} = -\frac{\bar{N}_x R}{2} \left(\frac{A_{11}}{D_{22}}\right)^{1/2} \quad (2-3)$$

$$\eta_s = \frac{\left(A_{12} + \frac{A_{33}}{2}\right)}{\sqrt{A_{11} A_{22}}}$$

$$\eta_p = \frac{(D_{12} + 2 D_{33})}{\sqrt{D_{11} D_{22}}}$$

$$\gamma = \frac{D_{11} A_{11}}{D_{22} A_{22}}$$

$$\beta = \left(\frac{m}{n}\right) \left(\frac{\pi R}{L}\right) \left(\frac{A_{22}}{A_{11}}\right)^{1/4}$$

$$\alpha = \frac{L^2}{2 R m^2 \pi^2 A_{22} \left(\frac{D_{22}}{A_{11}}\right)^{1/2}}$$

(2-3)
cont'd

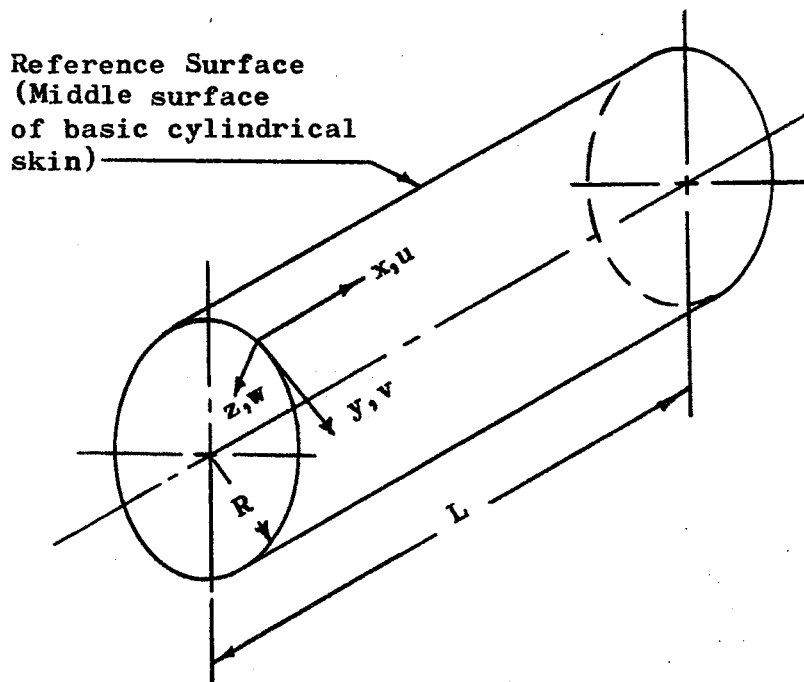


Figure 1 - Coordinate System

The various A_{ij} 's, D_{ij} 's, and C_{ij} 's of equations (2-2) and (2-3) are very important fundamental constants. The physical significance of these constants is discussed in Volume I [1]. The A_{ij} 's and D_{ij} 's are usually referred to as elastic constants while the C_{ij} 's might be identified as eccentricity coupling constants.

2.2 AXISYMMETRIC MODE

In order to apply equations (2-1) and (2-2) to practical structures, both axisymmetric and non-axisymmetric (checkerboard) buckling modes must be considered. Specialization to the axisymmetric case is achieved by allowing β to increase without bound ($\beta \rightarrow \infty$). This yields the following result:

$$\lim_{\beta \rightarrow \infty} \left(\bar{N}_{THIEL} \right)_c = \frac{\gamma}{4\alpha} + \alpha \left[1 - \frac{C_{12}}{2\alpha A_{22} (D_{22}/A_{11})^{1/2}} \right]^2 \quad (2-4)$$

By substituting equations (2-3) into (2-4), one can then obtain the following equation for axisymmetric buckling:

$$\left(\bar{N}_x \right)_c = \frac{m^2 \pi^2 D_{11}}{L^2} + \frac{L^2}{m^2 \pi^2 R^2 A_{22}} \left[1 - \frac{m^2 \pi^2}{L^2} C_{12} R \right]^2 \quad (2-5)$$

It is easily shown that, for $m > 0$, the second derivative of $(\bar{N}_x)_c$ with respect to m is always positive. Hence, a relative minimum for $(\bar{N}_x)_c$ can be located by differentiating equation (2-5) with respect to m and equating the result to zero. It is found that the derivative vanishes when

$$m = \frac{L}{\pi} \left[Z_1 \right]^{1/4} \quad (2-6)$$

where

$$Z_1 = \frac{1}{R^2 A_{22} \left(D_{11} + C_{12}^2 / A_{22} \right)} \quad (2-7)$$

The quantity Z_1 will always be a positive number. The four roots to equation (2-6) can therefore be expressed as follows:

$$\begin{aligned} m_1 &= + \left(\frac{L}{\pi} \right) \sqrt[4]{Z_1} \\ m_2 &= - \left(\frac{L}{\pi} \right) \sqrt[4]{Z_1} \\ m_3 &= + (i) \left(\frac{L}{\pi} \right) \sqrt[4]{Z_1} \\ m_4 &= - (i) \left(\frac{L}{\pi} \right) \sqrt[4]{Z_1} \end{aligned} \tag{2-8}$$

For applications to actual structures, only the single, real, positive root, m_1 , is of interest. For cases where $m_1 \geq 1$, the critical loading for axisymmetric buckling can be found by making the substitution $m = m_1$ in equation (2-5). The condition $m < 1$ is a physical impossibility for the problem under discussion. Therefore, whenever $m_1 < 1$, the critical loading for axisymmetric buckling is found by making the substitution $m = 1$ in equation (2-5).

Detailed derivations of the foregoing relationships are given in reference 15 which also presents the background information concerning the digital computer program of SECTION 6. This program makes use of the above equations in the computation of output listings and the generation of automatically plotted critical strains.

2.3 NON-AXISYMMETRIC MODE

To obtain the critical loading for non-axisymmetric buckling, equation (2-1) must be minimized with respect to two wave-type parameters. In this volume, α and β were selected for this purpose although m and n would have been equally suitable. Therefore, the partial derivative of equation

(2-1) was taken with respect to α and the result was set equal to zero.

From this it was found that the derivative will vanish whenever

$$\alpha = \left\{ \frac{[1 + 2\eta_p \sqrt{\gamma} \beta^2 + \gamma \beta^4][1 + 2\eta_s \beta^2 + \beta^4]}{4\beta^8} + U^2 \right\}^{1/2} \quad (2-9)$$

where

$$U = \frac{(C_{11} + C_{22})}{2(A_{22}D_{22})^{1/2}\beta^2} + \frac{C_{12}}{2A_{22}(D_{22}/A_{11})^{1/2}} + \frac{C_{21}}{2(A_{11}D_{22})^{1/2}\beta^4} \quad (2-10)$$

Here again, only the single, positive root is of interest to the problem under discussion. No theoretical consideration was given to the question of insuring that this α corresponds to a relative minimum rather than a relative maximum or an inflection point. Such safeguards were incorporated in the numerical procedures into which these relationships were injected. In addition, the minimization with respect to β is accomplished solely by numerical procedures. The digital computer program of SECTION 6 accomplishes this by screening over practical β values in small increments of β , always retaining the lowermost loading encountered. Constraints are imposed which disallow buckle patterns for which $m < 1$ and/or n is greater than zero but less than a set value selected by the analyst (usually unity).

A detailed derivation of equation (2-9) is given in reference 15 which, as already noted, also presents background information concerning the digital computer program of SECTION 6.

2.4 PREFERRED MATHEMATICAL FORMS

In order to better understand the basic equations of this volume, and to facilitate the presentation of design data in the most practical manner, it is helpful to rewrite the equations in different forms from those presented in the foregoing. Reference 15 gives all the detailed mathematical steps involved in these transformations. Only the major results are given here.

$$\frac{1}{E} \left[\left(\bar{N}_x \right)_c A_{11} \right] = \frac{\pi^2}{\left(\frac{R}{\sqrt{D_{22} A_{22}}} \right)^2 \left(\frac{L}{R} \right)^2} \left(\frac{m}{\beta^2} \right)^2 + \frac{2\pi^2}{\left(\frac{A_{33}}{A_{11}} \right)^{1/2} \left(\frac{A_{33}}{A_{22}} \right)^{1/2} \left(\frac{L}{\sqrt{A_{33} (D_{12} + 2D_{33})}} \right)^2} \left(\frac{m}{\beta} \right)^2 + \frac{\pi^2}{\left(\frac{L}{\sqrt{D_{11} A_{11}}} \right)^2} m^2$$

$$+ \frac{4\pi^2}{\left(\frac{L}{R} \right)^2} (m^2)(\beta^4) \frac{\left[\frac{1}{2\pi^2} \left(\frac{L}{R} \right)^2 \frac{(A_{33}/A_{22})^{1/2}}{(A_{33}/A_{11})^{1/2}} \left(\frac{1}{m^2} \right) - \frac{1}{2} \frac{(C_{11} + C_{22})}{R} \left(\frac{1}{\beta^2} \right) - \left(\frac{C_{12}}{2R} \right) \frac{(A_{33}/A_{22})^{1/2}}{(A_{33}/A_{11})^{1/2}} - \left(\frac{C_{21}}{2R} \right) \frac{(A_{33}/A_{11})^{1/2}}{(A_{33}/A_{22})^{1/2}} \left(\frac{1}{\beta^4} \right) \right]^2}{\left[1 + 2 \left(\frac{A_{12}}{A_{11}} \right)^{1/2} \left(\frac{A_{12}}{A_{22}} \right)^{1/2} \beta^2 + \left(\frac{A_{33}}{A_{11}} \right)^{1/2} \left(\frac{A_{33}}{A_{22}} \right)^{1/2} \beta^2 + \beta^4 \right]} \quad (2-11)$$

$$\frac{1}{E} \left[\left(\bar{N}_x \right)_c A_{11} \right] = \frac{\pi^2}{\left(\frac{R}{\rho_y} \right)^2 \left(\frac{L}{R} \right)^2} \left(\frac{m}{\beta^2} \right)^2 + \frac{2\pi^2}{\left(\frac{E}{G} \right) \left(\frac{\bar{t}_x}{t} \right)^{1/2} \left(\frac{\bar{t}_y}{t} \right)^{1/2} \left(\frac{L}{\rho_{xy}} \right)^2} \left(\frac{m}{\beta} \right)^2 + \frac{\pi^2}{\left(\frac{L}{\rho_x} \right)^2} m^2$$

$$+ \frac{4\pi^2}{\left(\frac{L}{R} \right)^2} (m^2)(\beta^4) \frac{\left[\frac{1}{2\pi^2} \left(\frac{L}{R} \right)^2 \frac{(\bar{t}_y/t)^{1/2}}{(\bar{t}_x/t)^{1/2}} \left(\frac{1}{m^2} \right) - \frac{1}{2} \frac{(C_{11} + C_{22})}{R} \frac{1}{\beta^2} - \frac{C_{12}}{2R} \frac{(\bar{t}_y/t)^{1/2}}{(\bar{t}_x/t)^{1/2}} - \frac{C_{21}}{2R} \frac{(\bar{t}_x/t)^{1/2}}{(\bar{t}_y/t)^{1/2}} \left(\frac{1}{\beta^4} \right) \right]^2}{\left[1 - \nu \frac{(\bar{t}_y/t)^{1/2}}{(\bar{t}_x/t)^{1/2}} \beta^2 + \left(\frac{E}{G} \right) \left(\frac{\bar{t}_x}{t} \right)^{1/2} \left(\frac{\bar{t}_y}{t} \right)^{1/2} \beta^2 + \beta^4 \right]} \quad (2-12)$$

2.4.1 GENERAL

One type of useful reformulation of the basic buckling equation (2-1) can be achieved through the substitution of equations (2-3) and subsequent algebraic manipulation. This leads to equation (2-11) which provides the basis for the STIFF option computations of digital computer program 4267 (see SECTION 6).

To obtain the additional reformulation expressed as equation (2-12), it is helpful to define the following quantities:

$$\begin{aligned}\rho_x &= \sqrt{D_{11} A_{11}} \\ \rho_y &= \sqrt{D_{22} A_{22}} \\ \rho_{xy} &= \sqrt{A_{33} (D_{12} + 2D_{33})}\end{aligned}\tag{2-13}$$

Recognizing the physical basis [1] for the elastic constants D_{ij} and A_{ij} , it is clear that the first two of equations (2-13) are quite logically selected as effective local radii of gyration in the axial and circumferential directions. The last of these three equations is an artificial, contrived definition which has been made in the interest of convenience. It should be noted, however, that this equation does display some similarity-of-form to the equations for ρ_x and ρ_y . It was this likeness which led to the use here of the notation ρ_{xy} . Additional clarification of the terms involved in equations (2-13) can be obtained from a study of Table III and its related notes.

In addition to the foregoing, equation (2-12) is also based on the use of the following simplified formulas for the elastic constants of interest:

$$\begin{aligned}
 A_{11} &= \frac{1}{Et_x} & D_{11} &= EI_x \\
 A_{22} &= \frac{1}{Et_y} & D_{22} &= EI_y \\
 A_{33} &= \frac{1}{Gt} & A_{12} = A_{21} &= \frac{-\nu}{Et_x} (B_{A12})
 \end{aligned} \tag{2-14}$$

Table III and its notes furnish helpful clarifying information concerning the quantities which appear in these expressions. Permissible values for B_{A12} are bounded as follows:

$$0 \leq B_{A12} \leq 1 \tag{2-15}$$

As noted in reference 15, little error is introduced if this quantity is taken equal to 0.50 for all cases. Therefore, this value was selected for use in the present volume. This practice, together with the substitution of equations (2-13) and (2-14) into equation (2-11), permits one to rewrite the basic buckling equation in the instructive form of equation (2-12). This expression provides the basis for the RATIO option of digital computer program 4267 (see SECTION 6).

2.4.2 AXISYMMETRIC MODE

The procedures described in SECTION 2.4.1 were likewise applied to rewrite equations (2-5), (2-6), and (2-7) in alternative, preferred mathematical forms. The following expressions were obtained for use in the RATIO option computations of digital computer program 4267 (see SECTION 6):

$$\frac{\sigma}{E} = \left[\left(\bar{N}_x \right)_c A_{11} \right] = \frac{\pi^2}{\left(\frac{L}{\rho_x} \right)^2} m^2 + \left(\frac{1}{\pi^2} \right) \left(\frac{L}{R} \right)^2 \frac{\left(\frac{\bar{t}_y}{t} \right) \left(\frac{1}{m^2} \right)}{\left(\frac{\bar{t}_x}{t} \right)} \left[1 - \pi^2 \left(\frac{C_{12}}{R} \right) \frac{m^2}{\left(\frac{L}{R} \right)^2} \right]^2 \quad (2-16)$$

where

$$m = \left[\frac{1}{\frac{\pi^4}{\left(\frac{L}{R} \right)^2} \frac{\left(\frac{\bar{t}_x}{t} \right)}{\left(\frac{\bar{t}_y}{t} \right)} \frac{1}{\left(\frac{L}{\rho_x} \right)^2} + \frac{\pi^4}{\left(\frac{L}{R} \right)^4} \left(\frac{C_{12}}{R} \right)^2} \right]^{1/4} \quad (2-17)$$

Since equations (2-5), (2-6), and (2-7) are already in a suitable form for the STIFF option computations, no additional transformations of these equations are required.

2.4.3 NON-AXISYMMETRIC MODE

The procedures described in SECTION 2.4.1 were likewise applied to rewrite equations (2-9) and (2-10) in alternative, preferred mathematical forms. Equations (2-18) and (2-19) were obtained for use in the RATIO option computations of digital computer program 4267 (see SECTION 6). Equations (2-20) and (2-21) were obtained for use in the STIFF option computations of the program.

2.5 SPECIAL GEOMETRICAL CONSIDERATIONS

The large number of geometrical values appearing in the foregoing equations makes it necessary to introduce a number of simplifying assumptions to arrive at a suitable format for general instability design curves. Several such simplifications have been selected based on practical considerations relative to the cross sections which lie in planes normal to the axis of revolution. A portion of such a section is depicted in Figure 2. It should be noted that b denotes the circumferential stringer

$$m = \left[\left(\frac{1}{4\pi^4} \right) \left(\frac{L}{R} \right)^4 \frac{(\bar{t}_y/t)}{(\bar{t}_x/t)} \left(\frac{R}{\rho_y} \right)^2 \left\{ \frac{1 + \frac{2}{\left(\frac{L}{\rho_{xy}} \right)^2 \left(\frac{E}{G} \right) \left(\frac{\bar{t}_x}{t} \right)^{1/2} \left(\frac{\bar{t}_y}{t} \right)^{1/2} \left(\frac{R}{\rho_y} \right)^2 \left(\frac{L}{R} \right)^2 (\beta^2)}{\left(\frac{L}{\rho_x} \right)^2 \left(\frac{R}{\rho_y} \right)^2 \left(\frac{L}{R} \right)^2 (\beta^4)} \right. \right. \\ \left. \left. \frac{\left[1 - \nu \frac{(\bar{t}_y/t)^{1/2}}{(\bar{t}_x/t)^{1/2}} (\beta^2) + \left(\frac{E}{G} \right) \left(\frac{\bar{t}_x}{t} \right)^{1/2} \left(\frac{\bar{t}_y}{t} \right)^{1/2} \beta^2 + \beta^4 \right] + 4 U^2 \beta^8}{4\beta^8} \right\}^{1/4} \right] \quad (2-18)$$

where,

$$U = \left(\frac{R}{\rho_y} \right) \left[\left(\frac{1}{2} \right) \left(\frac{C_{11}}{R} + \frac{C_{22}}{R} \right) \left(\frac{1}{\beta^2} \right) + \left(\frac{1}{2} \right) \left(\frac{C_{12}}{R} \right) \frac{(\bar{t}_y/t)^{1/2}}{(\bar{t}_x/t)^{1/2}} + \left(\frac{1}{2} \right) \left(\frac{C_{21}}{R} \right) \frac{(\bar{t}_x/t)^{1/2}}{(\bar{t}_y/t)^{1/2}} \left(\frac{1}{\beta^4} \right) \right] \quad (2-19)$$

$$M = \left[\left(\frac{1}{4\pi^4} \right) \left(\frac{L}{R} \right)^4 \left(\frac{A_{11}}{A_{22}} \right) \frac{R^2}{(D_{22}A_{22})} \left\{ \frac{4\beta^8}{\left[1 + \frac{(2)(D_{12} + 2D_{33})(A_{11}A_{22})^{1/2}}{L^2} \frac{L^2}{(D_{22}A_{22})} (\beta^2) + \frac{(D_{11}A_{11})}{(D_{22}A_{22})} (\beta^4) \right]} \right. \right. \\ \left. \left. \frac{\left[1 + \frac{2A_{12}}{(A_{11}A_{22})^{1/2}} (\beta^2) + \frac{A_{33}}{(A_{11}A_{22})^{1/2}} \beta^2 + \beta^4 \right] + 4U^2 \beta^8} \right\} \right]^{1/4} \quad (2-20)$$

where,

$$U = \frac{R}{(D_{22}A_{22})^{1/2}} \left[\frac{(C_{11} + C_{22})}{2R \beta^2} + \left(\frac{1}{2} \right) \left(\frac{C_{12}}{R} \right) \left(\frac{A_{11}}{A_{22}} \right)^{1/2} + \left(\frac{1}{2} \right) \left(\frac{C_{21}}{R} \right) \left(\frac{A_{22}}{A_{11}} \right)^{1/2} \left(\frac{1}{\beta^4} \right) \right] \quad (2-21)$$

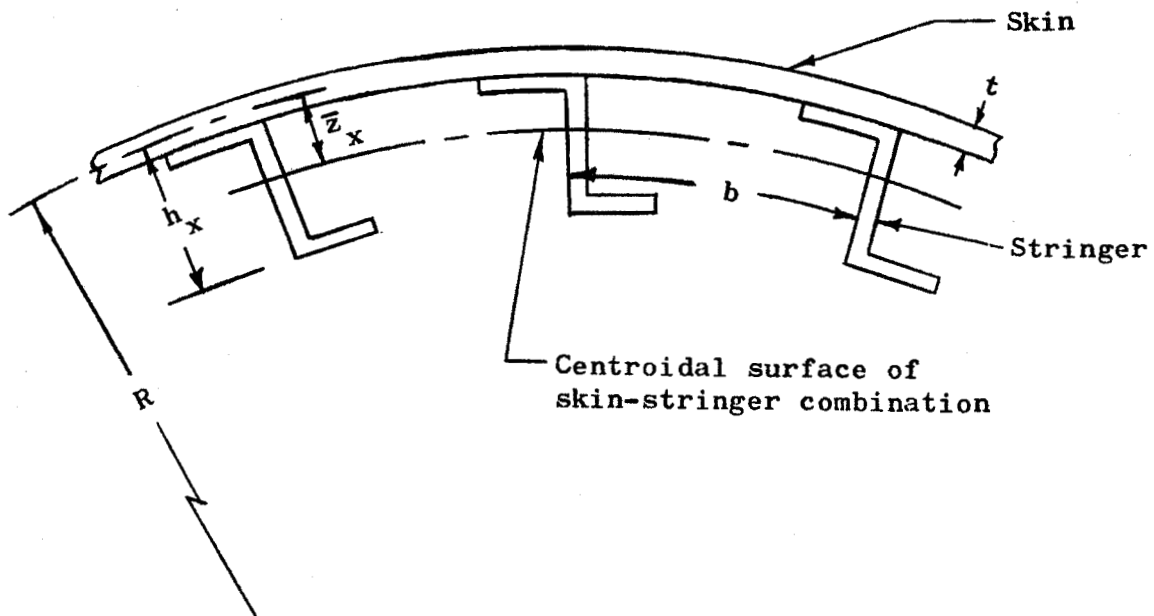


Figure 2 - Sample Wall Cross Section
Lying in Plane Normal to
Axis of Revolution

spacing. In addition, the area of a single stringer (no skin included) will be identified by the symbol A_s . The eccentricity is represented as \bar{z}_x . By assuming that

- (a) All of the skin and stringer material is fully effective and
- (b) The stringer centroid (no skin included) lies at its mid-height,

one can employ the simple laws of mechanics to arrive at the following formula for the absolute magnitude of the eccentricity:

$$|\bar{z}_x| = \frac{\left(\frac{1}{2}\right) \left(\frac{\bar{t}_x}{t} - 1\right) \left(h_x + \frac{t}{2}\right)}{\left(\frac{\bar{t}_x}{t}\right)} \quad (2-22)$$

where

$$\bar{t}_x = \left[\frac{A_s}{b} + t \right] \quad (2-23)$$

Introduction of the assumption that $h_x \gg (t/2)$ permits one to rewrite equation (2-22) as

$$|\bar{z}_x| = \frac{\left(\frac{1}{2}\right) \left(\frac{\bar{t}_x}{t} - 1\right) (h_x)}{\left(\frac{\bar{t}_x}{t}\right)} \quad (2-24)$$

It then becomes convenient to define two new quantities, C_{hx} and $C_{\rho x}$, as follows:

$$C_{hx} = \frac{|\bar{z}_x|}{h_x} \quad (2-25)$$

$$C_{\rho x} = \frac{\rho_x}{h_x} \quad (2-26)$$

The quantity ρ_x is the local centroidal radius of gyration of the skin-stringer combination. For practical skin-stringer configurations, the value $C_{\rho x} = .33$ will usually approximate the actual conditions reasonably well. Therefore, the buckling curves of SECTION 4 are based upon this value. These curves are also based upon the relationship

$$C_{hx} = \left(\frac{1}{2}\right) \frac{\left[\left(\frac{\bar{t}_x}{t}\right) - 1\right]}{\left(\frac{\bar{t}_x}{t}\right)} \quad (2-27)$$

which is obtained through the substitution of equation (2-25) into (2-24). The computations performed within digital computer program 4267 make use of the C_{hx} and $C_{\rho x}$ values to arrive at reasonable measures of eccentricity based on specified ρ_x values. That is, $|\bar{z}_x|$ is found from the following equation which results from the substitution of equation (2-26) into (2-25):

$$|\bar{z}_x| = \left(\frac{C_{hx}}{C_{\rho x}} \right) \rho_x \quad (2-28)$$

Although the buckling curves of SECTION 4 incorporate the several approximations indicated above, it should be noted that digital computer program 4267 features several options which allow one to obtain improved accuracy for given configurations. For example, one may choose to input actual C_{hx} and $C_{\rho x}$ values for selected structures. Furthermore, an option is provided which permits the insertion of primary input geometry in the form of elastic constants and eccentricity coupling constants.

Consideration will now be given to cross sections which lie in radial planes. A portion of such a section is depicted in Figure 3. The symbol

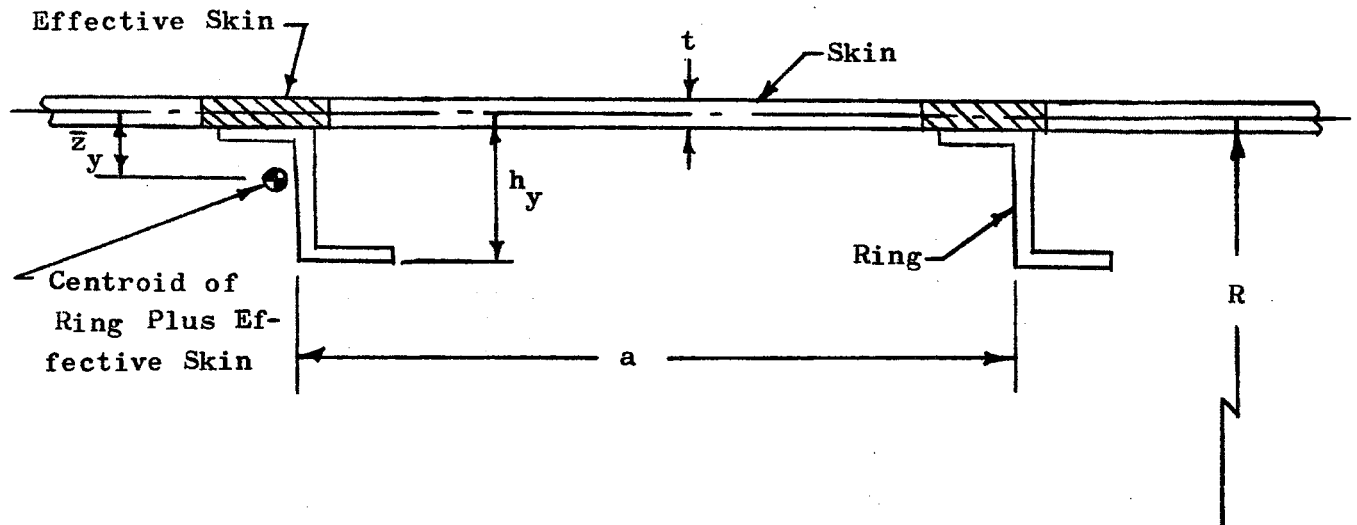


Figure 3 - Sample Wall Cross Section
Lying in Radial Plane

a denotes the spacing between rings while the eccentricity value of interest is identified as \bar{z}_y . Unlike the case for the section shown in Figure 2, the usual proportions encountered in skin-stringer-ring constructions make it improper to assume that the complete skin-ring combination is fully effective. Normally, only a relatively short length of skin can be considered to essentially behave as part of the ring cross section. Therefore, no relationship corresponding to equation (2-24) is involved in the ring-related simplifications embodied in the buckling curves of SECTION 4. These simplifications are introduced only through the terms C_{hy} and $C_{\rho y}$ which are defined as follows:

$$C_{hy} = \frac{|\bar{z}_y|}{h_y} \quad (2-29)$$

$$C_{\rho y} = \frac{\rho_y}{h_y} \quad (2-30)$$

The quantity ρ_y is the local centroidal radius of gyration of the ring-effective skin combination. Substitution of equation (2-30) into (2-29) permits one to express the absolute magnitude of the hoop eccentricity as follows:

$$|\bar{z}_y| = \left(\frac{C_{hy}}{C_{\rho y}} \right) \rho_y \quad (2-31)$$

To generate the buckling curves of SECTION 4, this equation was used to arrive at reasonable measures of $|\bar{z}_x|$ based upon the specified ρ_y . The values

$$\begin{aligned} C_{hy} &= .425 \\ C_{\rho y} &= .4 \end{aligned} \quad (2-32)$$

were used for this purpose. These were selected as reasonable estimates for the proportions likely to be found in practical skin-stringer-ring constructions. Note however that digital computer program 4267 features an

input format which allows one to obtain improved accuracy for given configurations. For example, one may choose to input actual C_{hy} and C_{py} values for selected structures. Furthermore, as already noted, an option is provided which permits the insertion of primary input geometry in the form of elastic constants and eccentricity coupling constants.

SECTION 3

ANALYSIS METHODS

The methods given in this volume deal only with general instability (see GLOSSARY, Volume I [1]). No consideration is given here to the panel instability mode (see GLOSSARY, Volume I [1]). The methods employ the smearing-out technique whereby discrete stiffness values are averaged over the entire surface of the cylinder. One must therefore exercise engineering judgement to prevent misapplication to configurations having excessively large stiffener spacings. In addition, it should be noted that only classical theoretical solutions are employed. Influences from initial imperfections are completely ignored. It is therefore recommended that the values obtained from this volume be reduced in accordance with the knock-down criterion of Volume V [12].

The methods presented here for the analysis of general instability basically consist of the following:

- (a) A collection of buckling curves (see SECTION 4) which embody a number of simplifications which are reasonable for conventional skin-stringer-ring configurations.
- (b) Digital computer program 4267 (see SECTION 6) which can be used to obtain single-point solutions and/or additional buckling curves. The input options of this program enable one to obtain particular solutions of greater accuracy than can be obtained from the curves of SECTION 4. These curves sacrifice accuracy in the interest of generality and practicality. The computer program also enables one to analyze corrugated and waffle configurations, neither of which are covered by the given buckling curves since all of these plots apply only to conventional skin-stringer-ring arrangements and proportions.

The simplifying assumptions embodied in the buckling curves of SECTION 4 are as follows:

- (a) The entire skin-stringer combination is assumed to be fully effective.
- (b) The stringer centroid (no skin included) is assumed to lie at its mid-height.
- (c) $h_x \gg \frac{t}{2}$

- (d) $C_{\rho x} = .33$
- (e) $C_{hy} = .425$
- (f) $C_{\rho y} = .4$
- (g) $D_{33} = 0$
- (h) $D_{12} = D_{21} = 0$
- (i) $C_{12} = C_{21} = 0$

Assumptions (h) and (i) are justified on the basis that the constants D_{12} , D_{21} , C_{12} , and C_{21} involve the available mechanisms for interplay which are dependent upon Poisson's-ratio influences. For usual stringer and ring arrangements, this interplay is of a minimal nature since it occurs primarily in the localized regions where the stiffeners intersect. However, when the stringer and ring spacings are very small (as for waffle configurations) and/or the contributions from stiffener rigidities approach that of the skin (also as for waffle configurations), non-zero values should be used for D_{12} , D_{21} , C_{12} and C_{21} . In such cases, one must resort to point-solutions using digital computer program 4267.

It is important to recall here that a Donnell-type theory furnishes the basis for the methods given in this volume. Therefore, these methods cannot be applied to cases of non-axisymmetric buckling where the number of circumferential full-waves is small. As a rule-of-thumb, it is suggested that the methods be considered inapplicable where

$$0 < n < 2 \quad (3-1)$$

All of the curves given in SECTION 4 completely ignore this restriction. Therefore, to insure that this condition is not violated, one should supplement the plotted data with appropriate checks from digital computer runs using program 4267. However, most practical configurations likely to be encountered will not display buckle patterns with $0 < n < 2$. Hence it should not prove necessary to make the suggested check for every configuration investigated. When a large number of candidate designs are to be

studied, it will usually be reasonable for one to assume that the wave-number restrictions are satisfied so that the foregoing check need only be made as a final operation for a few selected cases.

It is further suggested that, in applying the methods of this volume, one give some consideration to the influences which shallow rings and/or wide stringer spacings might have on the elastic constant A_{22} . For the axisymmetric buckling mode it seems clear that the A_{22} value should reflect the difference in flexibility between the discretely loaded ring of Figure 4(a) and its uniformly loaded counterpart shown in Figure 4(b). For this purpose, a correction factor C_R can be established which is simply a ratio of these two flexibilities.

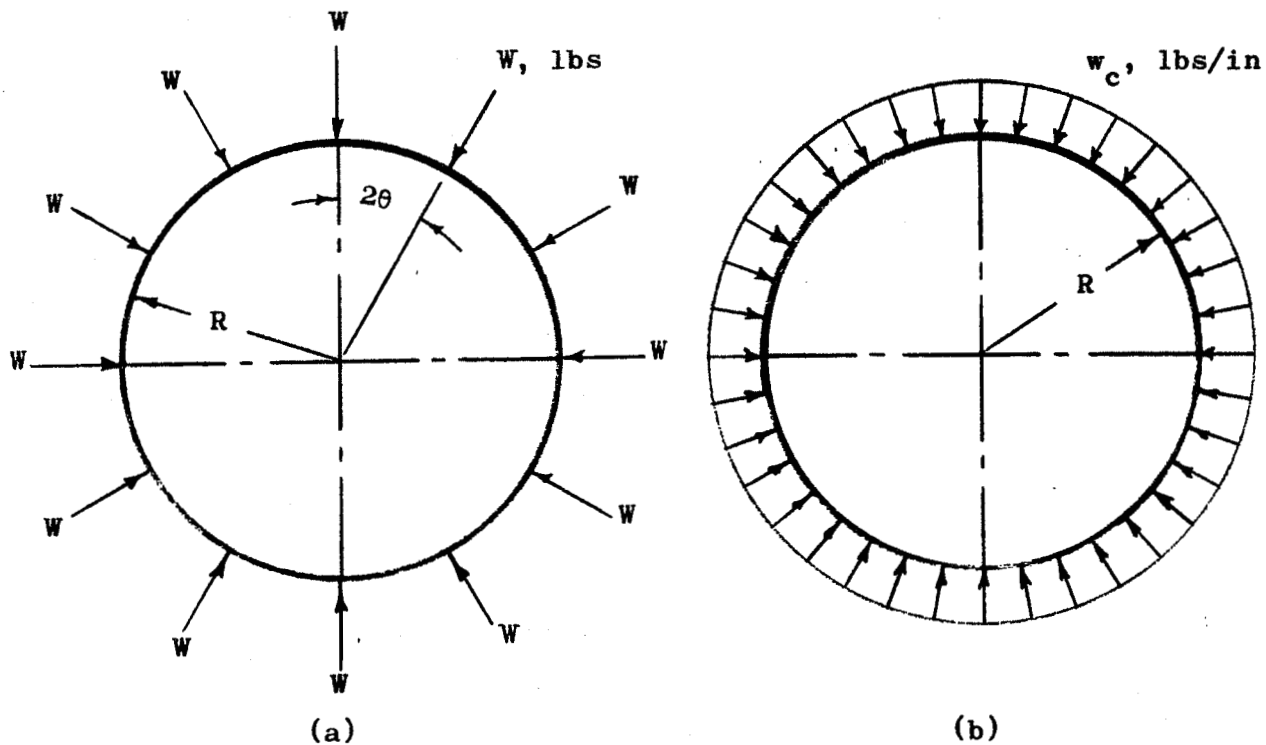


Figure 4 - Alternative Ring Loading Conditions

The following formula [16] can be used for the radial deflection of the discretely loaded ring:

$$\Delta_R = \frac{WR^3}{2EI_r} \left[\frac{1}{\sin^2 \theta} \left(\frac{\theta}{2} + \frac{\sin \theta \cos \theta}{2} \right) - \frac{1}{\theta} \right] \quad (3-2)$$

where

Δ_R = Radial deflection at points of load application.

θ = Half-angle between discrete load points, radians.

The radial deflection for the uniformly loaded case is denoted δ_R and the following formula for this quantity is easily derived:

$$\delta_R = \frac{w_c R^2}{A_r E} \quad (3-3)$$

To establish a basis for comparison between these two situations, the following relationship between W and w_c is employed:

$$W = \frac{2\pi R}{N_s} w_c \quad (3-4)$$

where

N_s = Number of discrete load points for the case of Figure 4(a); (Number of stringers)

The factor C_R may then be defined as follows:

$$C_R = \frac{\delta_R}{\Delta_R + \delta_R} = \frac{1}{\frac{\Delta_R}{\delta_R} + 1} \quad (3-5)$$

By substituting equations (3-2), (3-3), and (3-4) into equation (3-5), the following result is obtained:

$$C_R = \frac{1}{1 + \left(\frac{\pi K}{N_s}\right) \left(R^2 A_r / \bar{I}_r\right)} \quad (3-6)$$

where

$$K = \left[\frac{1}{\sin^2 \theta} \left(\frac{\theta}{2} + \frac{\sin \theta \cos \theta}{2} \right) - \frac{1}{\theta} \right] \quad (3-7)$$

and

$$\theta = \frac{\pi}{N_s} \quad (3-8)$$

Figure 5 presents plots of C_R versus $(R^2 A_r / \bar{I}_r)$ for selected constant values of N_s . The form of equation (3-7) leads to the usual numerical difficulties that one encounters when handling small differences between large numbers. Therefore, the curves of Figure 5 were plotted from digital computer results obtained to double-precision accuracy.

It is thought that the most logical use of the C_R value is as a correction factor to the quantity $(1/A_{22})$ appearing in any terms which do not vanish as $\beta \rightarrow \infty$. However, such corrections have not yet been incorporated into either the buckling curves of SECTION 4 or digital computer program 4267. Therefore, Figure 5 can only be used here as a means to check the applicability of the given methods. It is suggested that, when $C_R \geq 0.95$, one might conclude that the discreteness mechanism depicted in Figure 4 is of negligible importance. On the other hand, lower C_R values would indicate a need to apply engineering judgement in evaluating results obtained from the curves of SECTION 4 and digital computer program 4267. However, it will be found that, for most realistic skin-stringer-ring constructions, the geometric proportions will be such that C_R is essentially equal to unity.

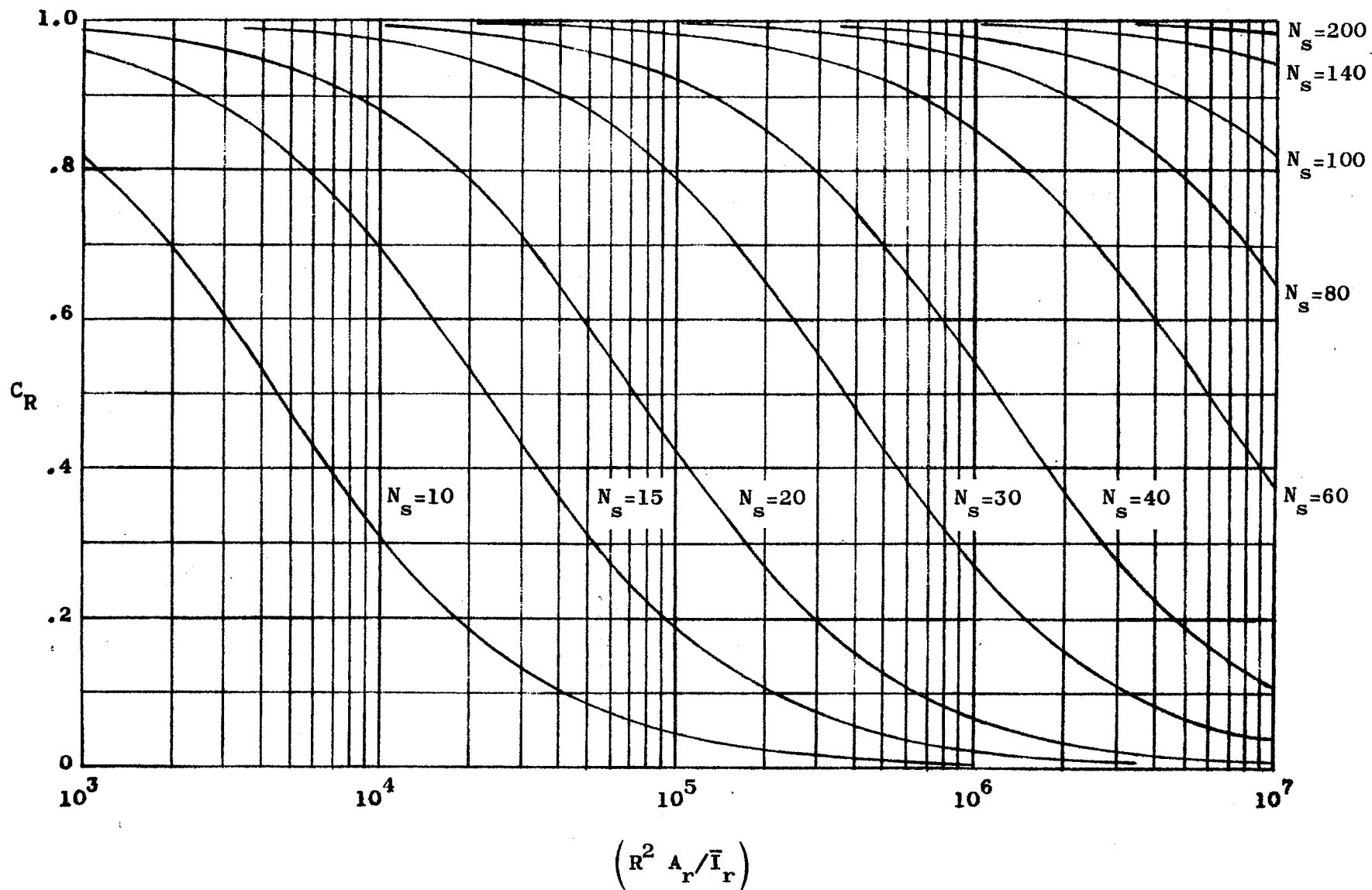


Figure 5 - C_R Factor

SECTION 4

BUCKLING CURVES

The curves of this section present critical strain values for axially compressed circular cylinders having eccentric stringers and rings. To make proper use of these curves, one should refer to the instructions furnished in SECTION 3, "ANALYSIS METHODS". All of these curves were developed by using digital computer program 4267 (see SECTION 6) in conjunction with an automatic plotting machine. The machine located individual points through which the curves were drawn by hand. Since the plotting machine does not have the capability to print out lower case letters, the symbols t , x , and the y of \bar{t}_y are shown in upper case notation.

The following information is furnished to clarify the meanings of all terms appearing on the plots:

- $T \text{ BAR } X = \bar{t}_x$ = Thickness of appropriate smeared-out area of cross sections lying in planes normal to the axis of revolution (see Table III and its notes).
- $T \text{ BAR } Y = \bar{t}_y$ = Thickness of appropriate smeared-out area of cross sections lying in radial planes (see Table III and its notes).
- $T = t$ = Thickness of basic cylindrical skin in conventional skin-stringer-ring constructions. To facilitate application to other configurations and to cases where buckling of the isotropic skin panels precedes general instability, it is helpful to note that this quantity enters into the given curves through both the elastic constant A_{33} and equation (2-27).

- L = Overall length of the cylinder. (Note that this quantity is NOT the ring spacing).
- R = Radius to middle surface of basic cylindrical skin.
- ρ_y = Local circumferential radius of gyration. This quantity may be computed from

$$\rho_y = \sqrt{\frac{\bar{I}_{r+s}}{A_{r+s}}} \quad \text{or} \quad \rho_y = \sqrt{\frac{\bar{I}_r}{A_r}} \quad (4-1)$$

as applicable (see Table III and its notes).

- RHO X = ρ_x = Local longitudinal radius of gyration. This quantity may be computed from the equation

$$\rho_x = \sqrt{\frac{\bar{I}_x}{\bar{t}_x}} \quad (4-2)$$

(See Table III and its notes).

- SIGMA BAR = $\bar{\sigma}$ = Critical compressive buckling stress obtained by dividing the critical longitudinal compressive running load by \bar{t}_x .

- E = Young's modulus.

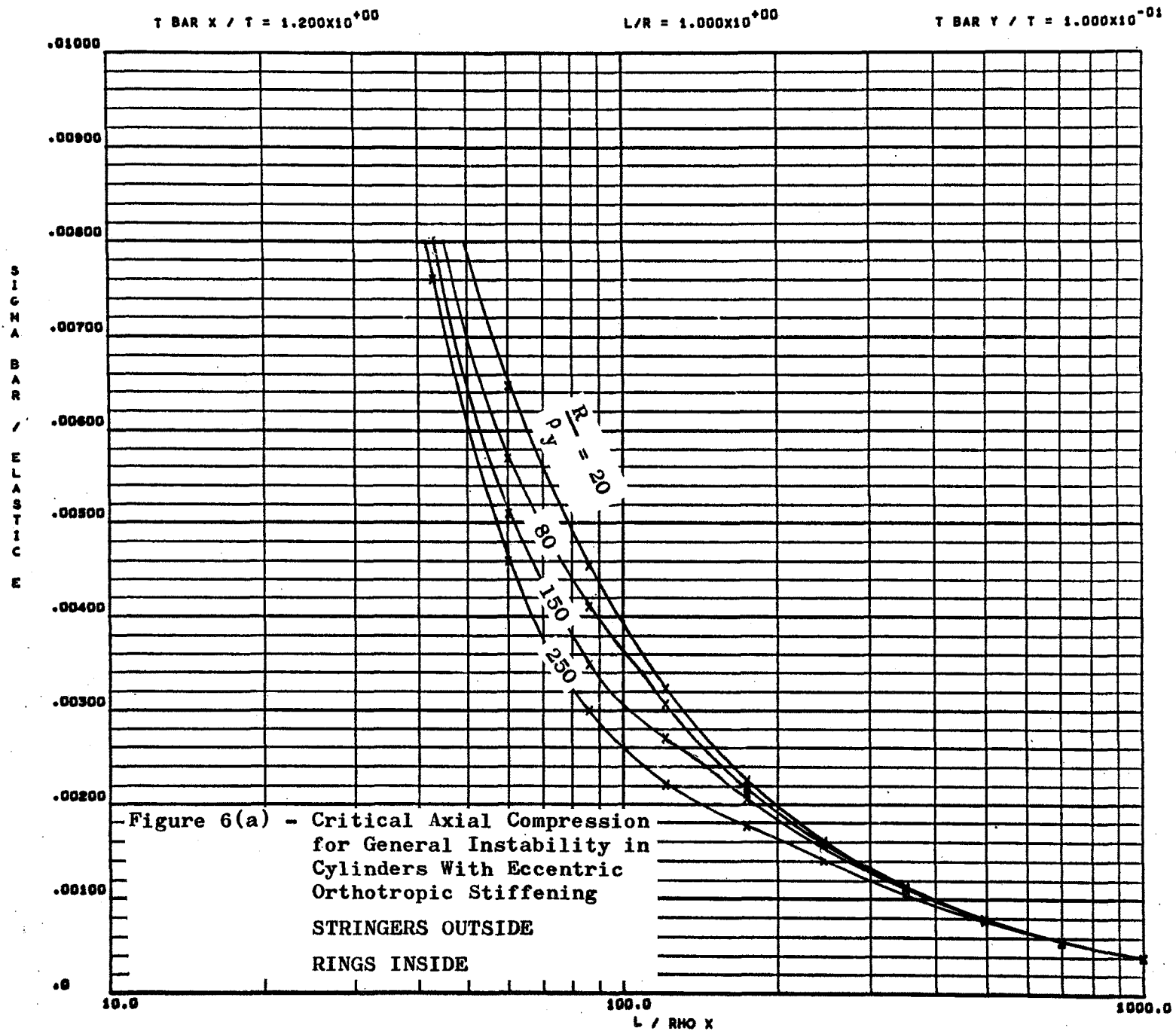
Table I lists the families of curves provided here for the case of external stringers and internal rings. Table II lists the families of curves provided here for the case of internal stringers and internal rings.

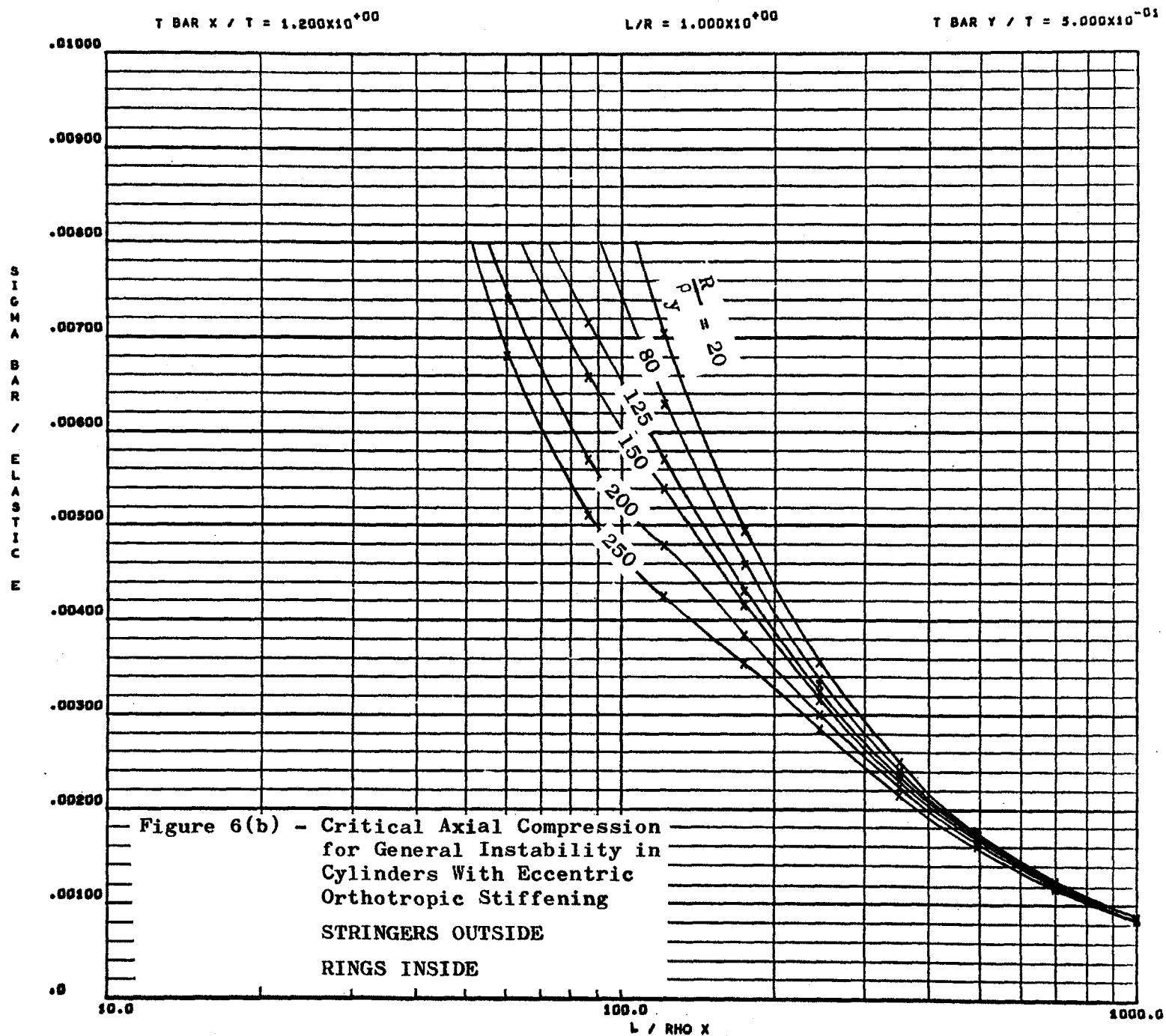
TABLE I - Table of Contents for Curves of
Critical Axial Compression for
General Instability in Cylinders
With Eccentric Orthotropic Stiffening
(External Stringers and Internal Rings)

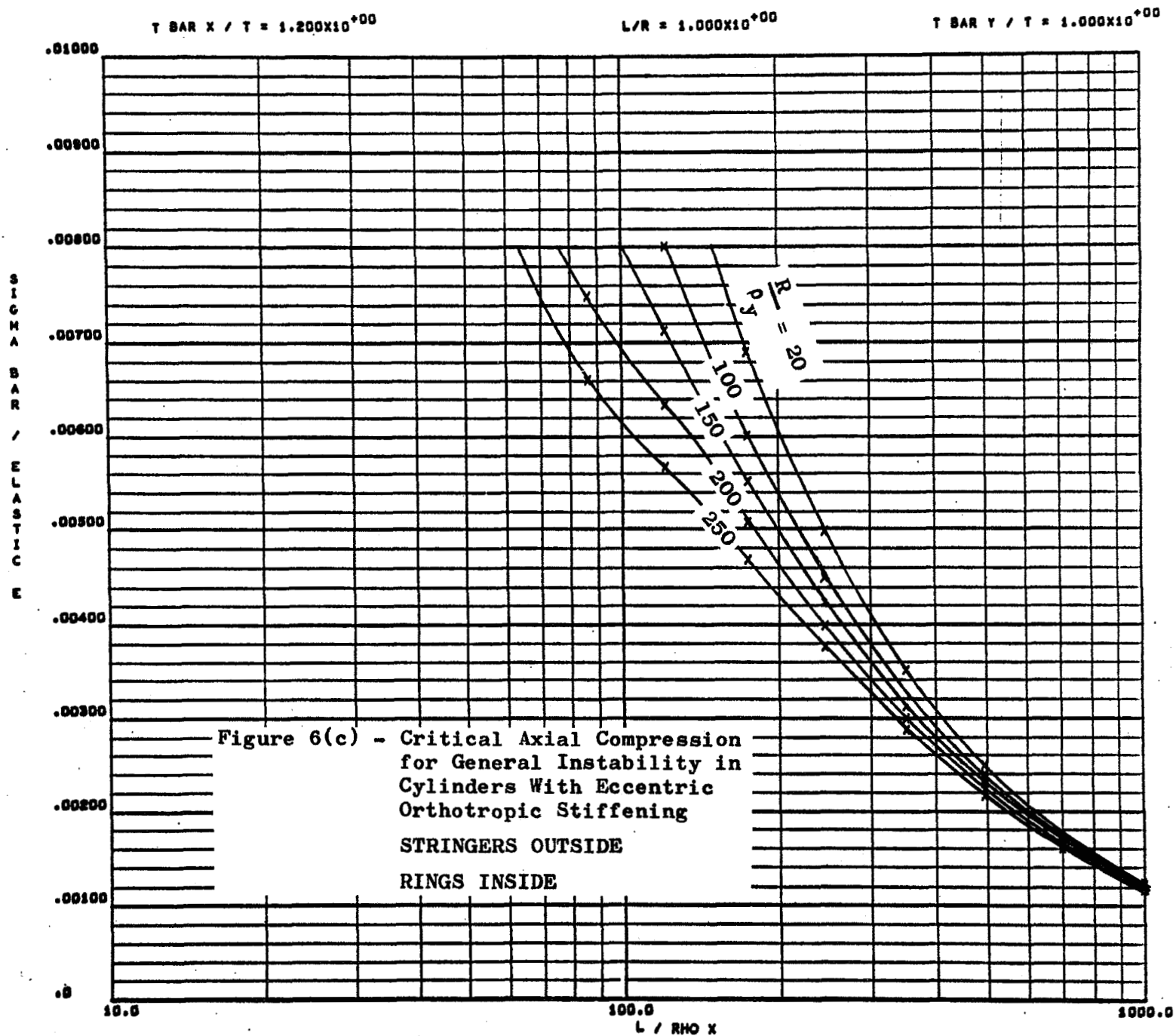
<u>Figure Number</u>	$\left(\frac{\bar{t}_x}{t}\right)$	$\left(\frac{L}{R}\right)$	$\left(\frac{\bar{t}_y}{t}\right)$	<u>Page</u>
6(a)	1.2	1.0	0.1	4-5
6(b)	1.2	1.0	0.5	4-6
6(c)	1.2	1.0	1.0	4-7
6(d)	2.0	1.0	0.1	4-8
6(e)	2.0	1.0	0.5	4-9
6(f)	2.0	1.0	1.0	4-10
6(g)	3.0	1.0	0.1	4-11
6(h)	3.0	1.0	0.5	4-12
6(i)	3.0	1.0	1.0	4-13
6(j)	1.2	2.0	0.1	4-14
6(k)	1.2	2.0	0.5	4-15
6(l)	1.2	2.0	1.0	4-16
6(m)	2.0	2.0	0.1	4-17
6(n)	2.0	2.0	0.5	4-18
6(o)	2.0	2.0	1.0	4-19
6(p)	3.0	2.0	0.1	4-20
6(q)	3.0	2.0	0.5	4-21
6(r)	3.0	2.0	1.0	4-22
6(s)	1.2	3.0	0.1	4-23
6(t)	1.2	3.0	0.5	4-24
6(u)	1.2	3.0	1.0	4-25
6(v)	2.0	3.0	0.1	4-26
6(w)	2.0	3.0	0.5	4-27
6(x)	2.0	3.0	1.0	4-28
6(y)	3.0	3.0	0.1	4-29
6(z)	3.0	3.0	0.5	4-30
6(aa)	3.0	3.0	1.0	4-31

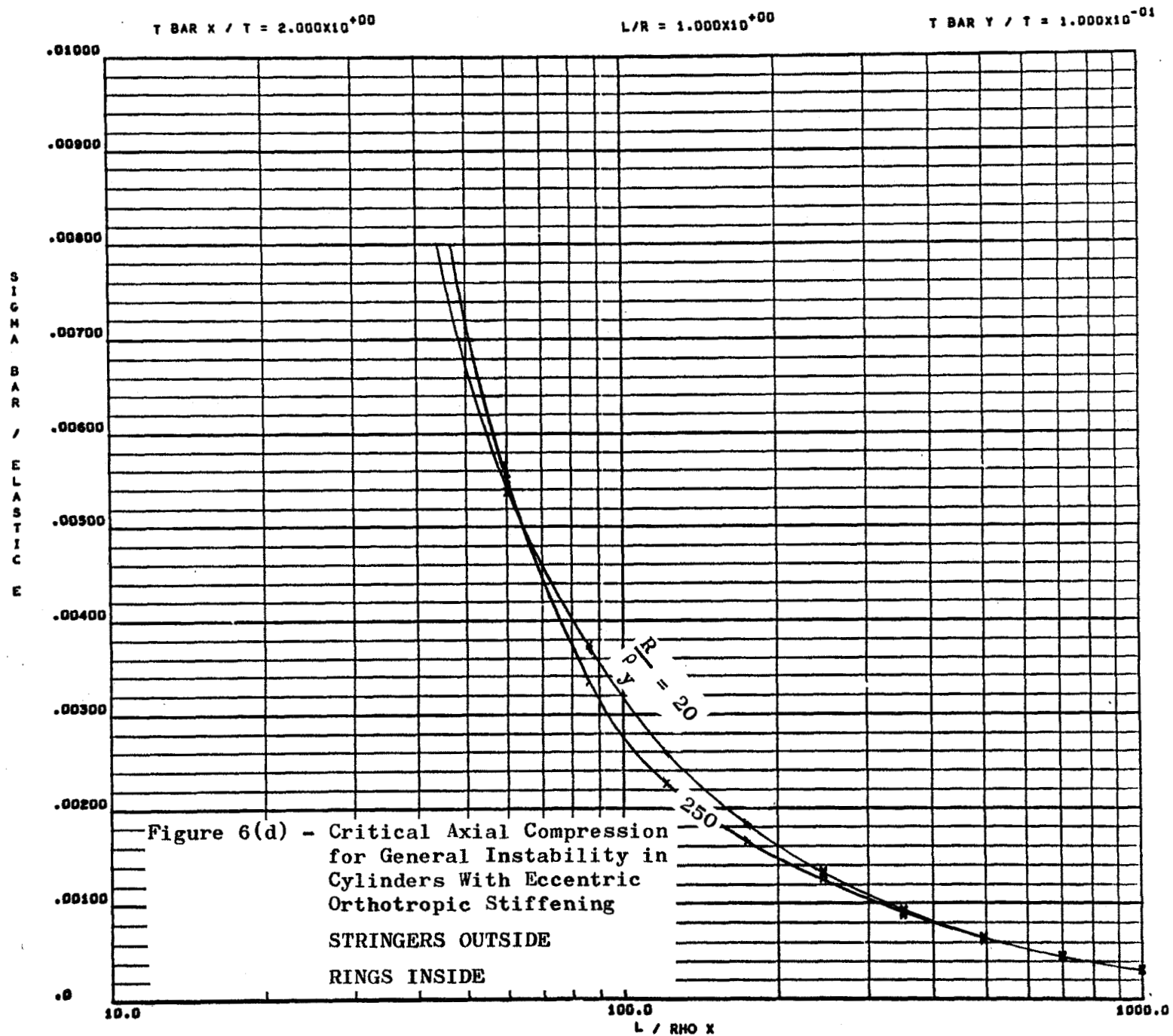
TABLE II - Table of Contents for Curves of Critical Axial Compression for General Instability in Cylinders With Eccentric Orthotropic Stiffening (Internal Stringers and Internal Rings)

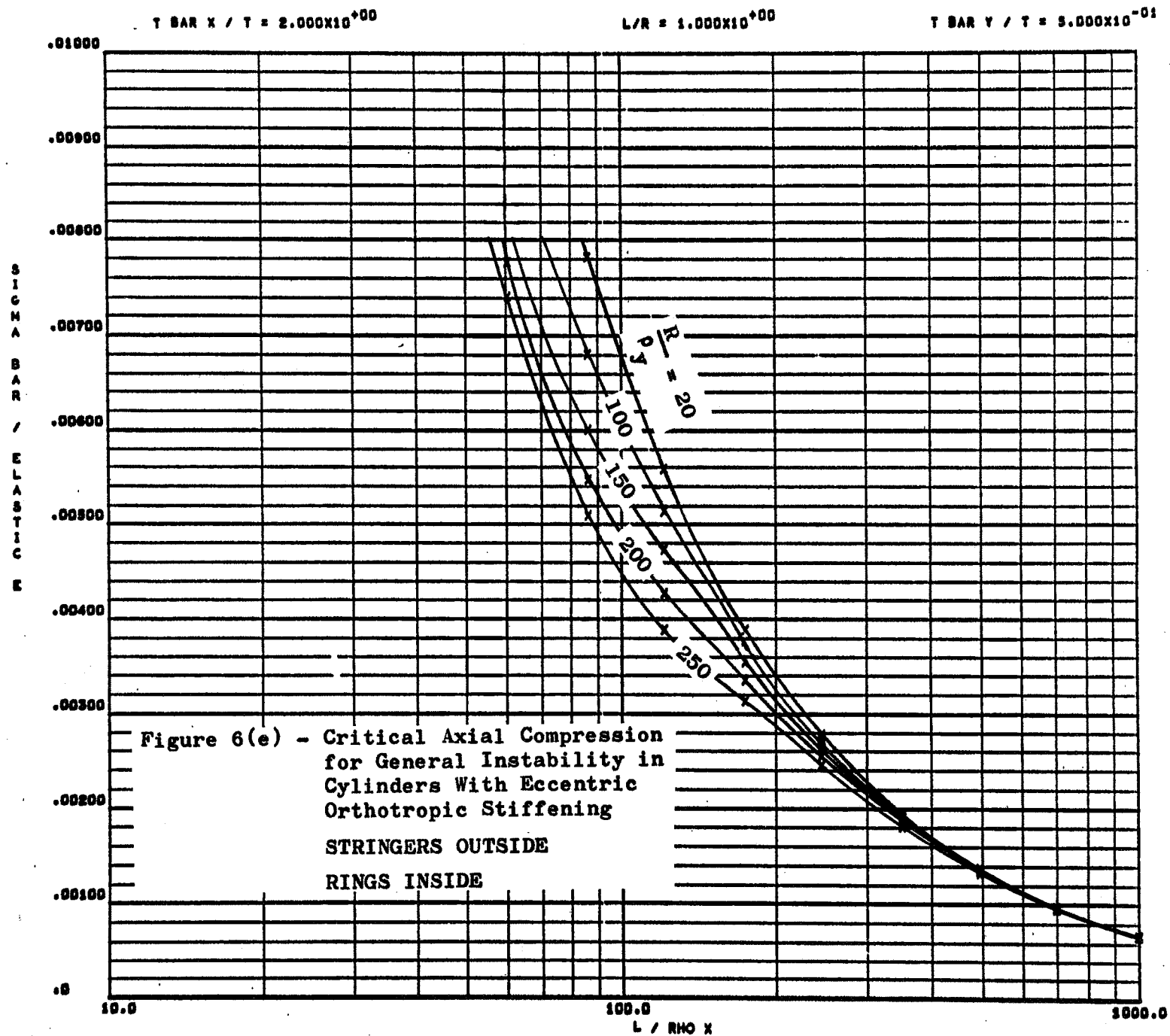
<u>Figure Number</u>	$\left(\frac{\bar{t}_x}{t}\right)$	$\left(\frac{L}{R}\right)$	$\left(\frac{\bar{t}_y}{t}\right)$	<u>Page</u>
7(a)	1.2	1.0	0.1	4-32
7(b)	1.2	1.0	0.5	4-33
7(c)	1.2	1.0	1.0	4-34
7(d)	2.0	1.0	0.1	4-35
7(e)	2.0	1.0	0.5	4-36
7(f)	2.0	1.0	1.0	4-37
7(g)	3.0	1.0	0.1	4-38
7(h)	3.0	1.0	0.5	4-39
7(i)	3.0	1.0	1.0	4-40
7(j)	1.2	2.0	0.1	4-41
7(k)	1.2	2.0	0.5	4-42
7(l)	1.2	2.0	1.0	4-43
7(m)	2.0	2.0	0.1	4-44
7(n)	2.0	2.0	0.5	4-45
7(o)	2.0	2.0	1.0	4-46
7(p)	3.0	2.0	0.1	4-47
7(q)	3.0	2.0	0.5	4-48
7(r)	3.0	2.0	1.0	4-49
7(s)	1.2	3.0	0.1	4-50
7(t)	1.2	3.0	0.5	4-51
7(u)	1.2	3.0	1.0	4-52
7(v)	2.0	3.0	0.1	4-53
7(w)	2.0	3.0	0.5	4-54
7(x)	2.0	3.0	1.0	4-55
7(y)	3.0	3.0	0.1	4-56
7(z)	3.0	3.0	0.5	4-57
7(aa)	3.0	3.0	1.0	4-58

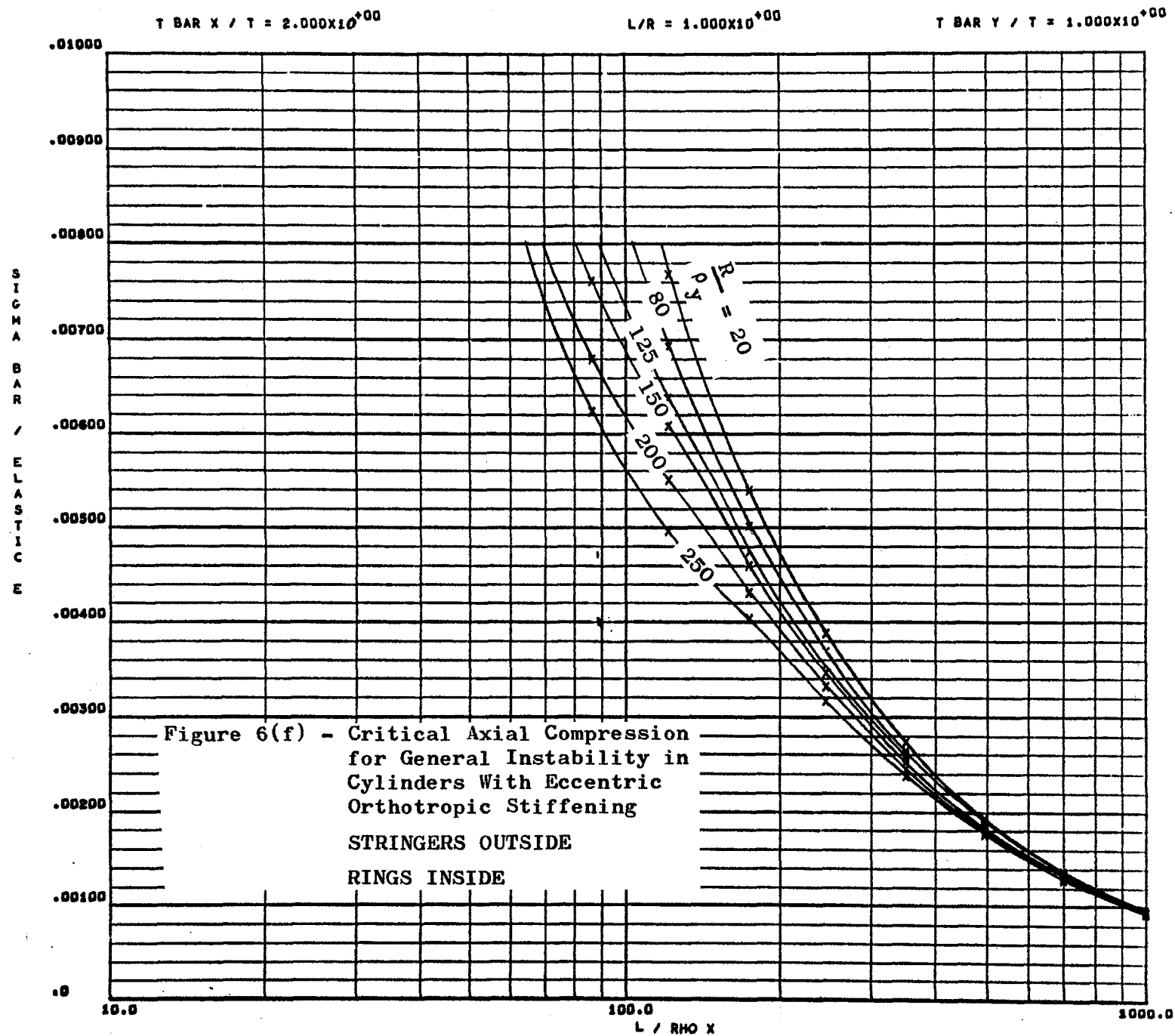


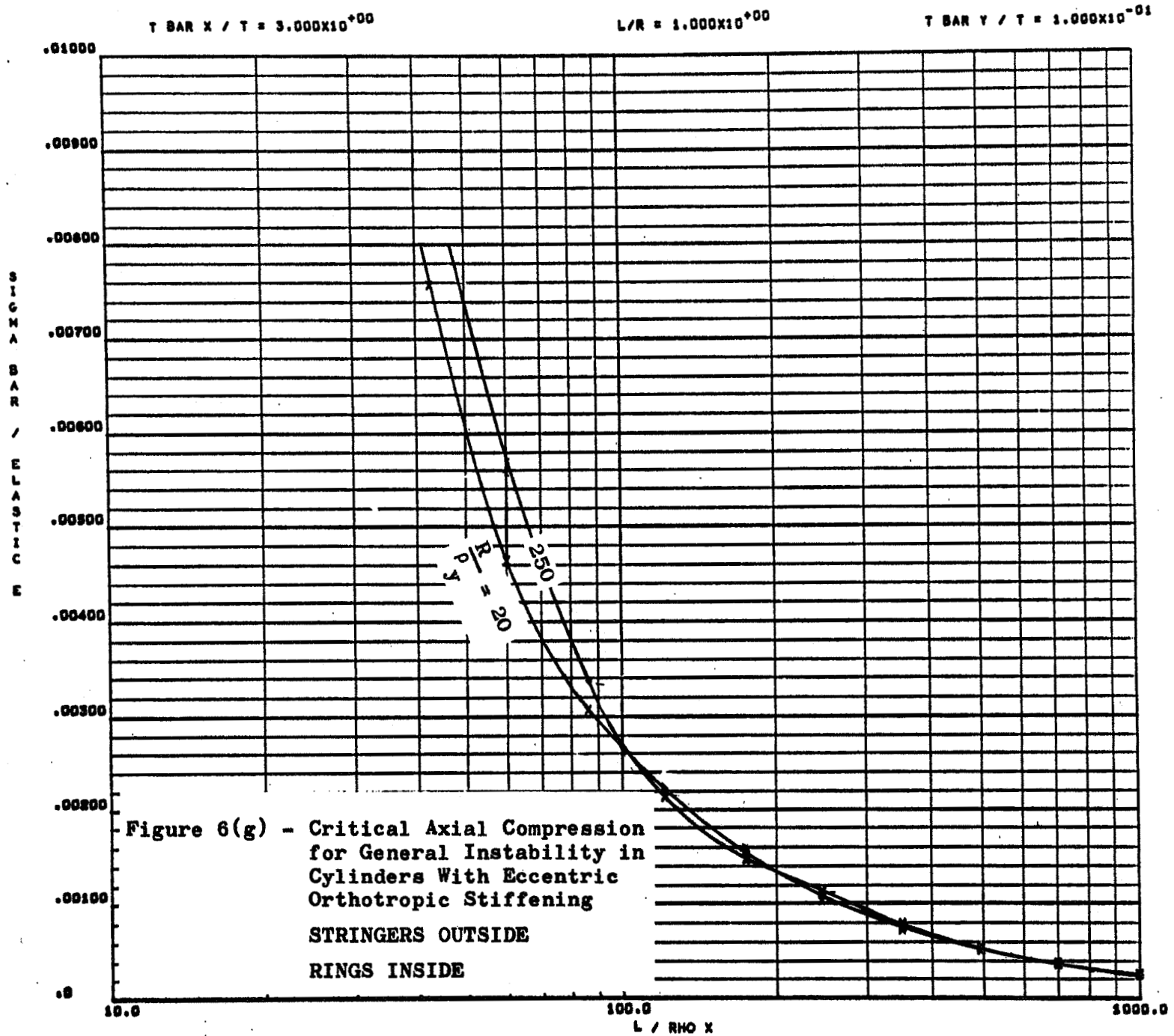


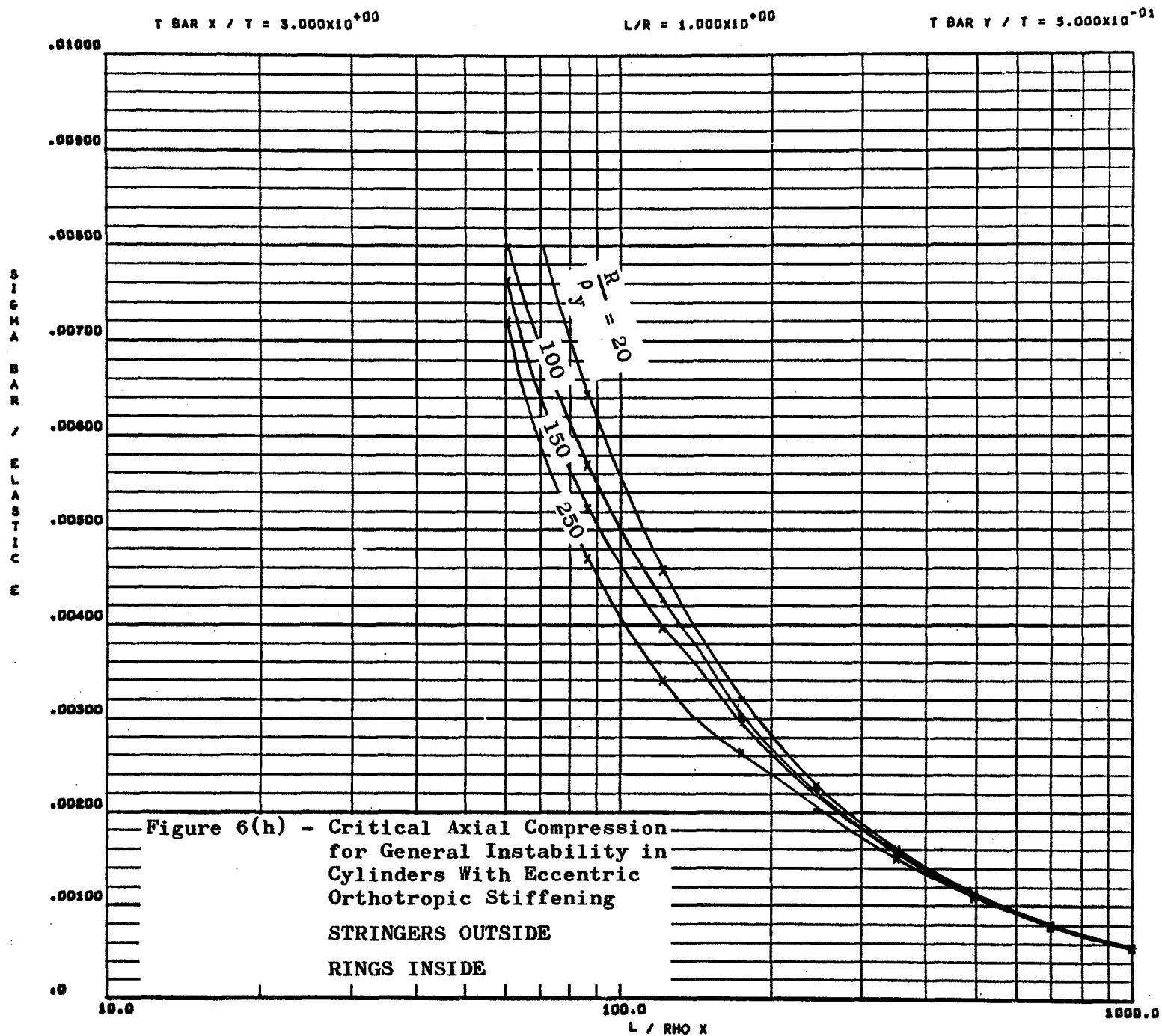


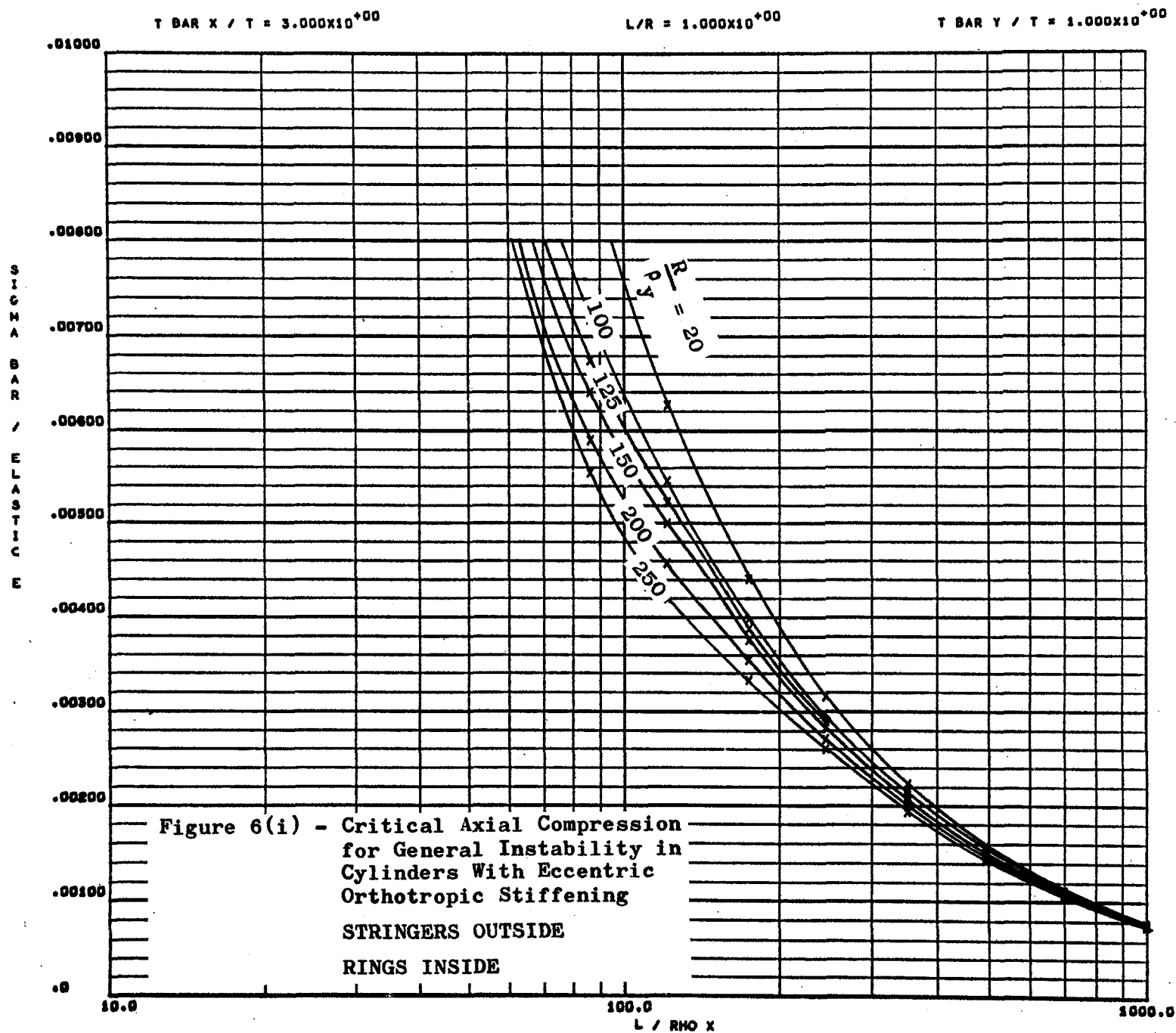


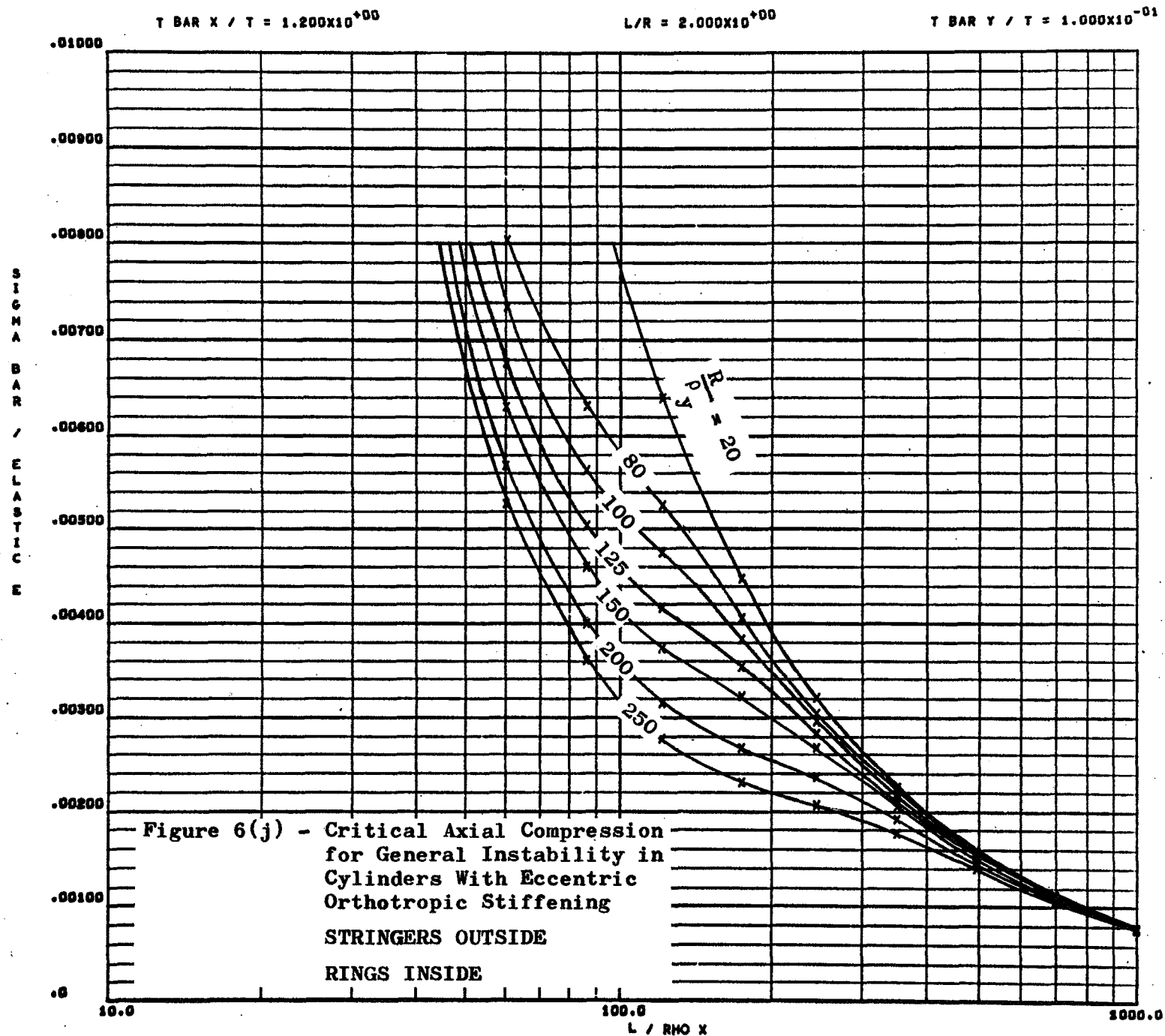


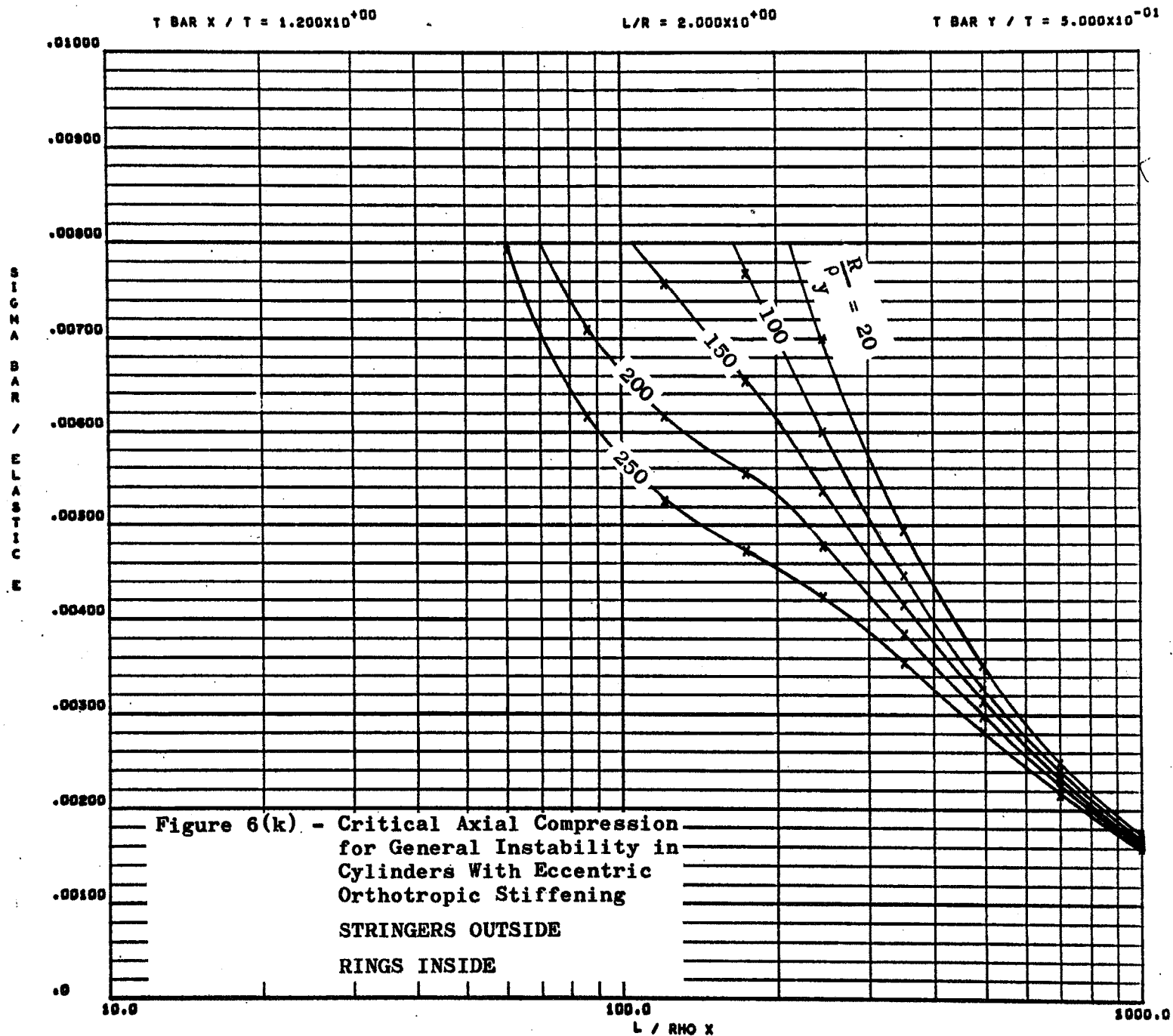


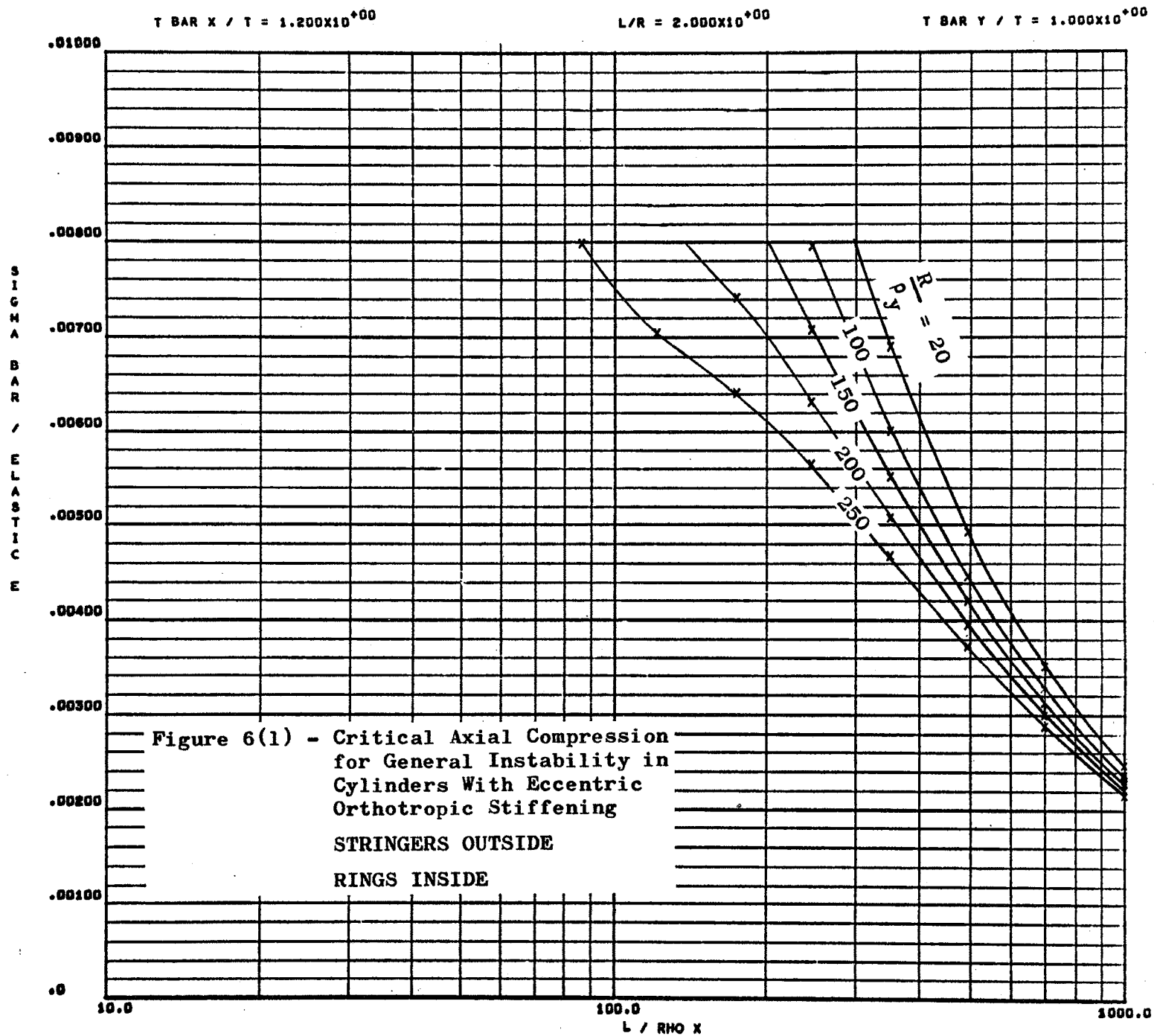


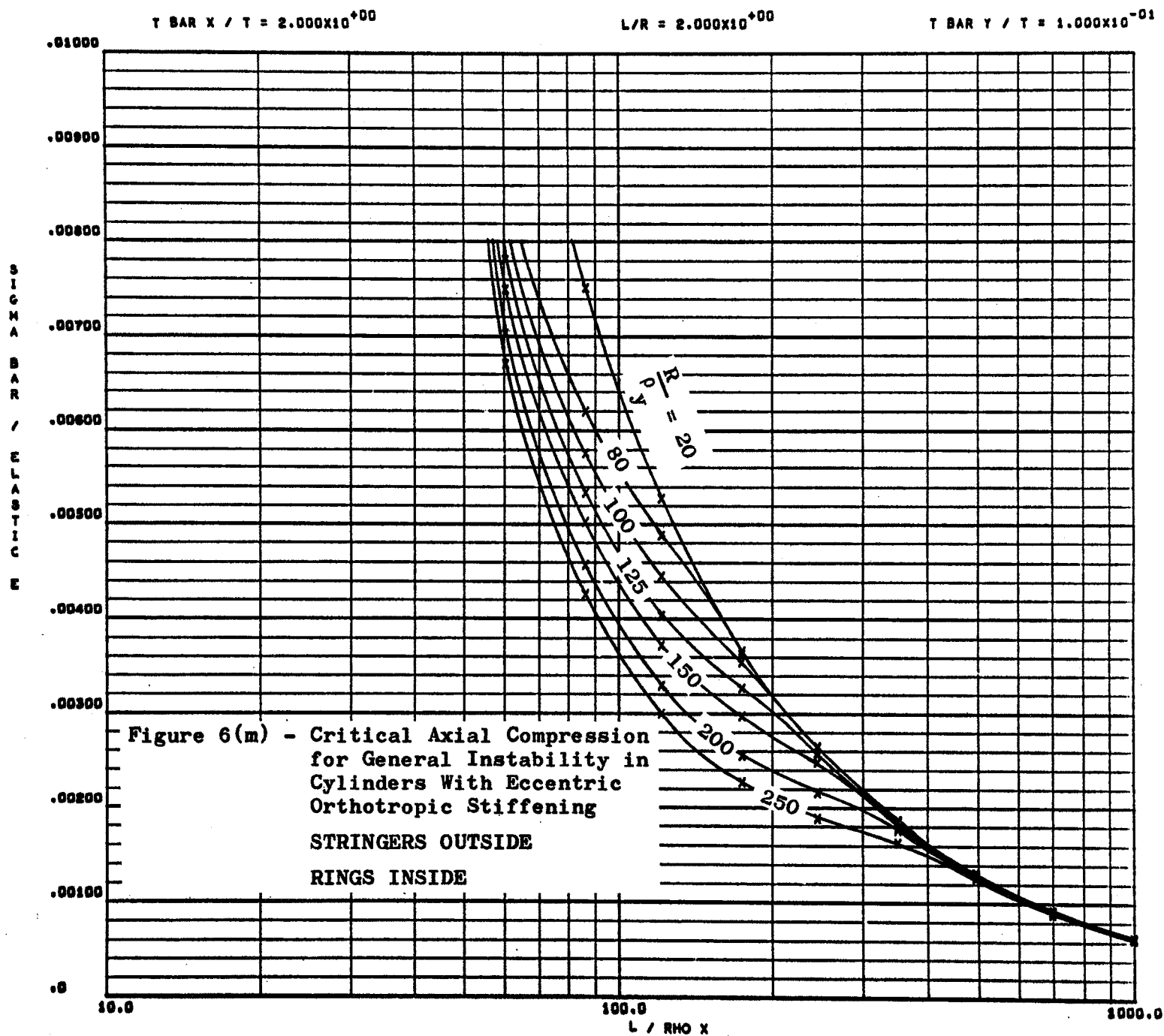


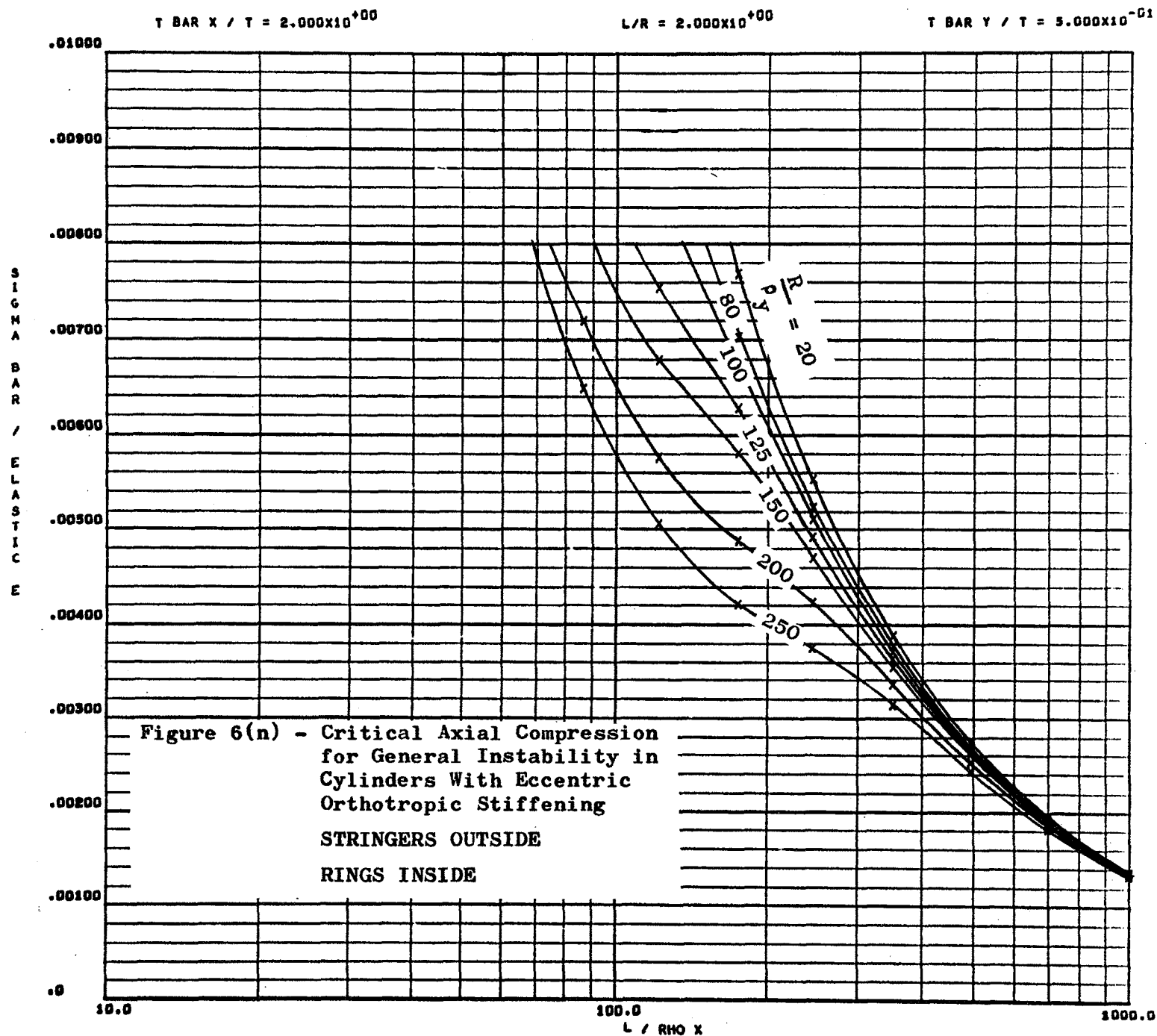


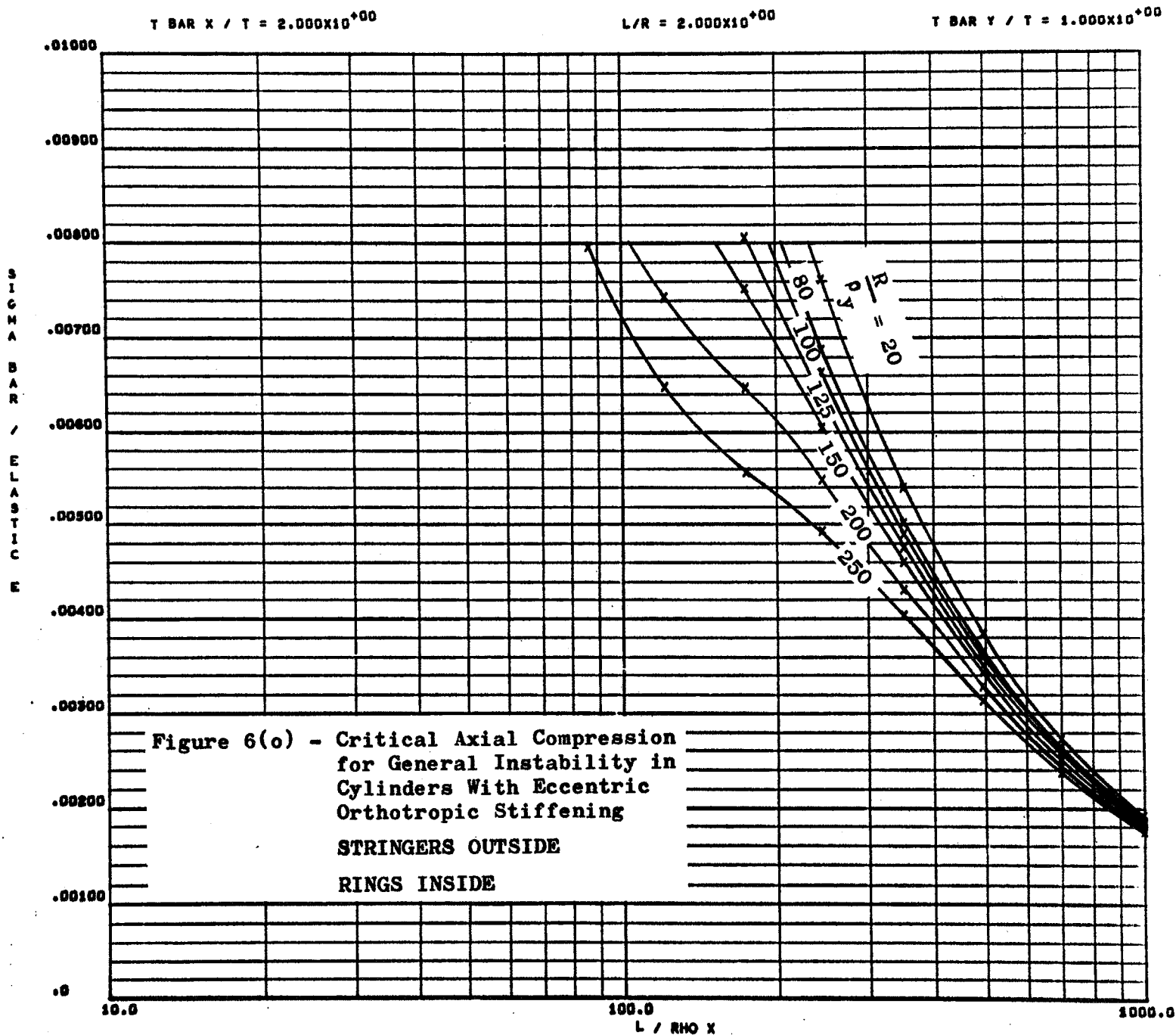


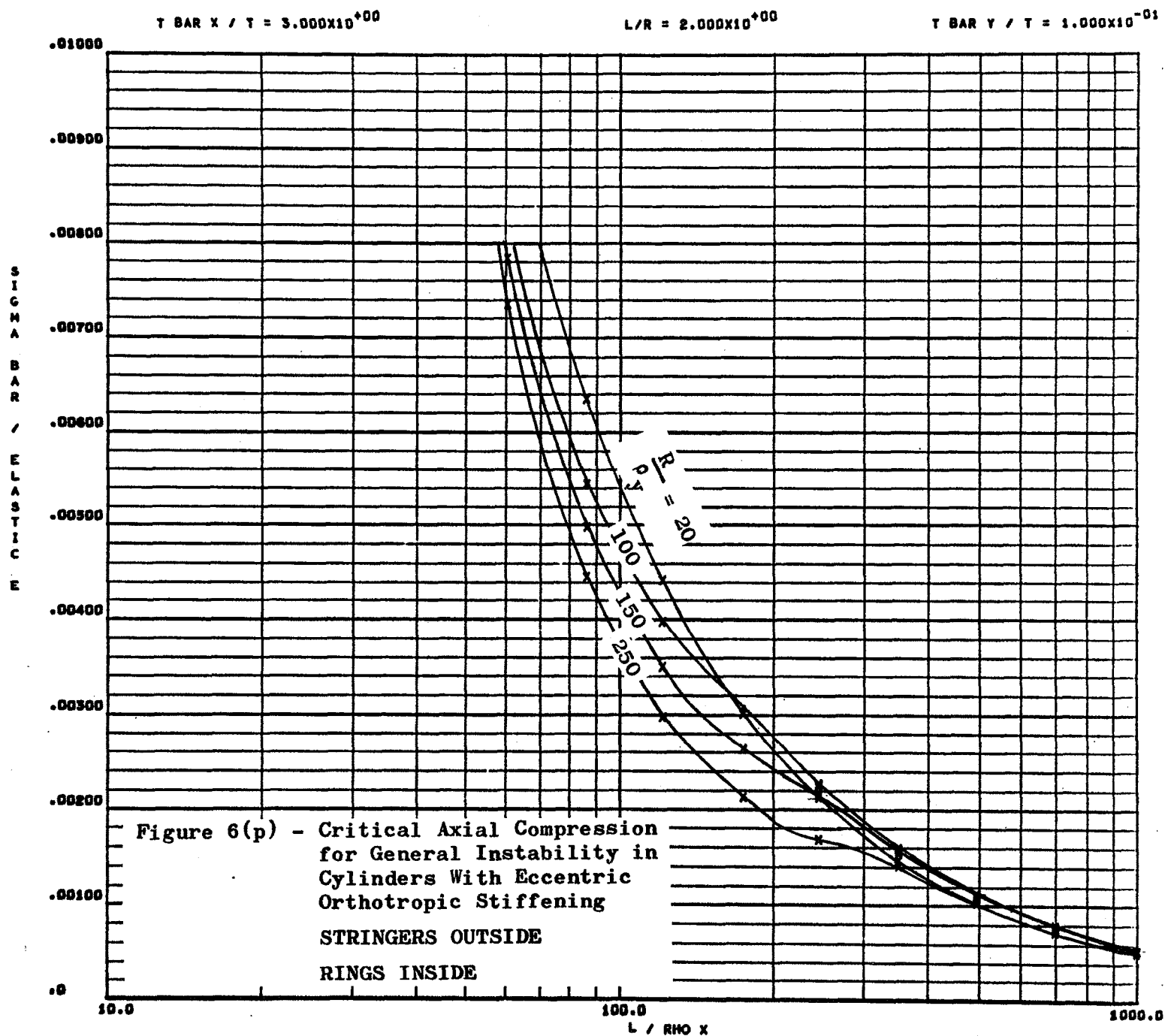


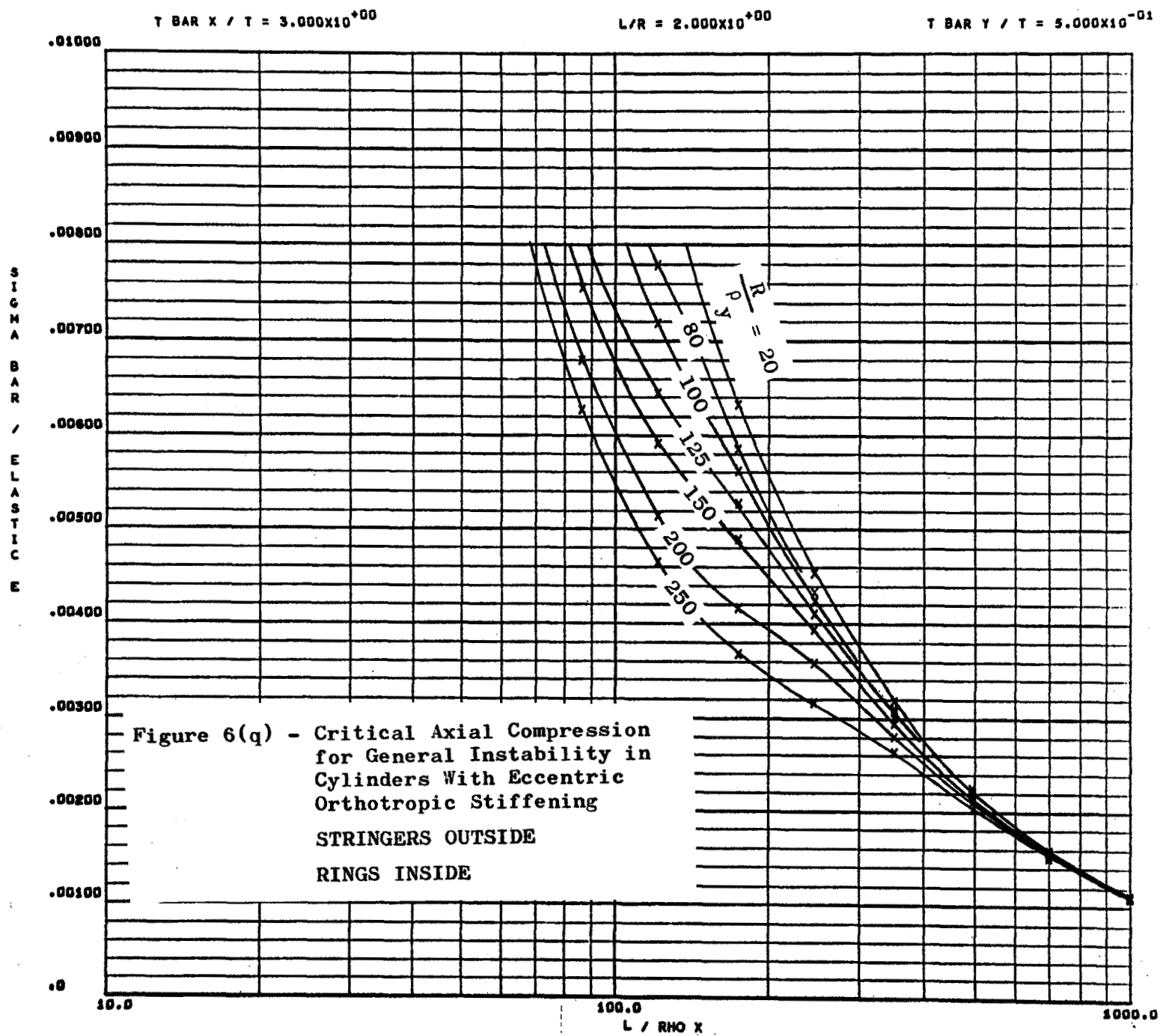


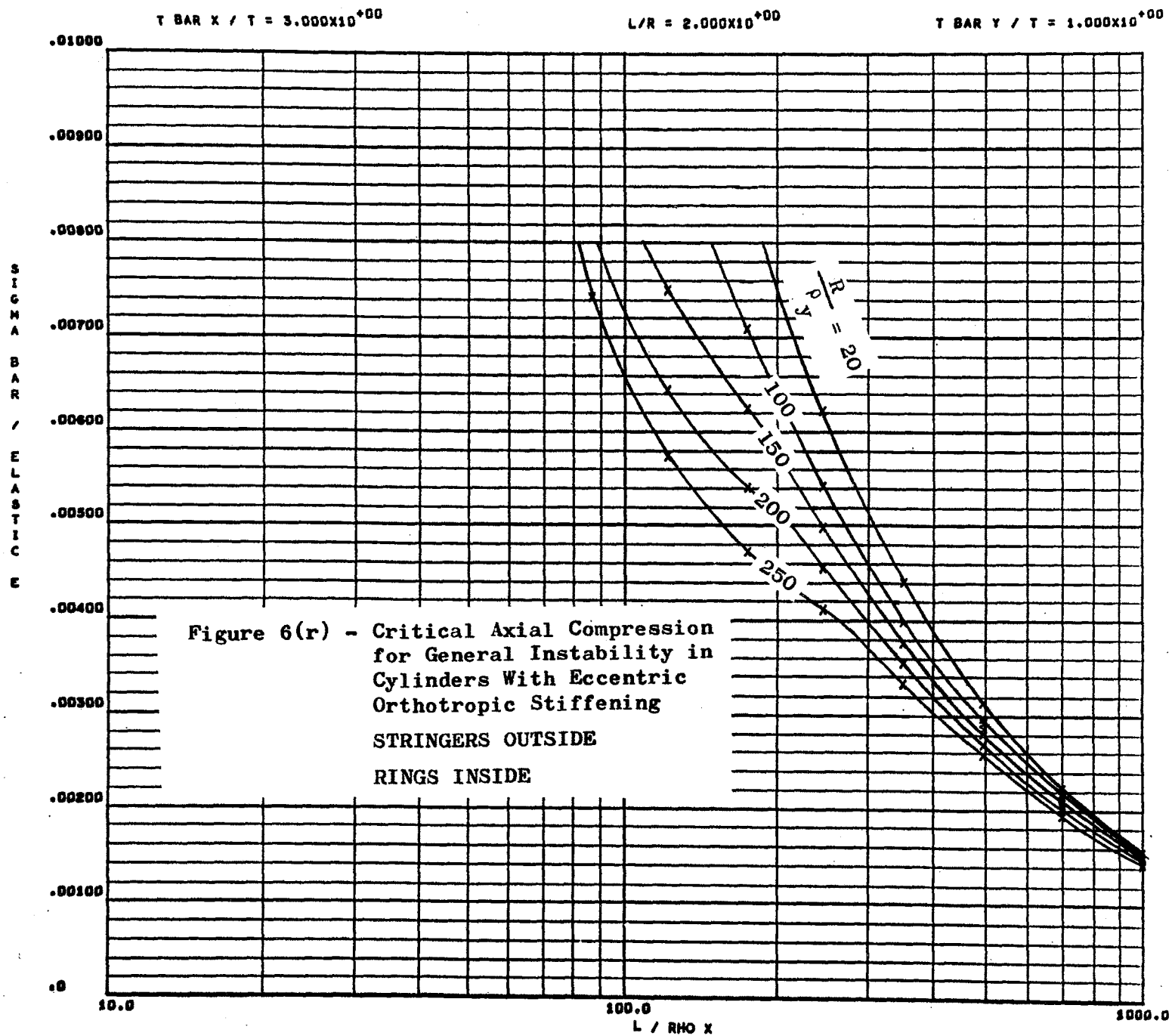


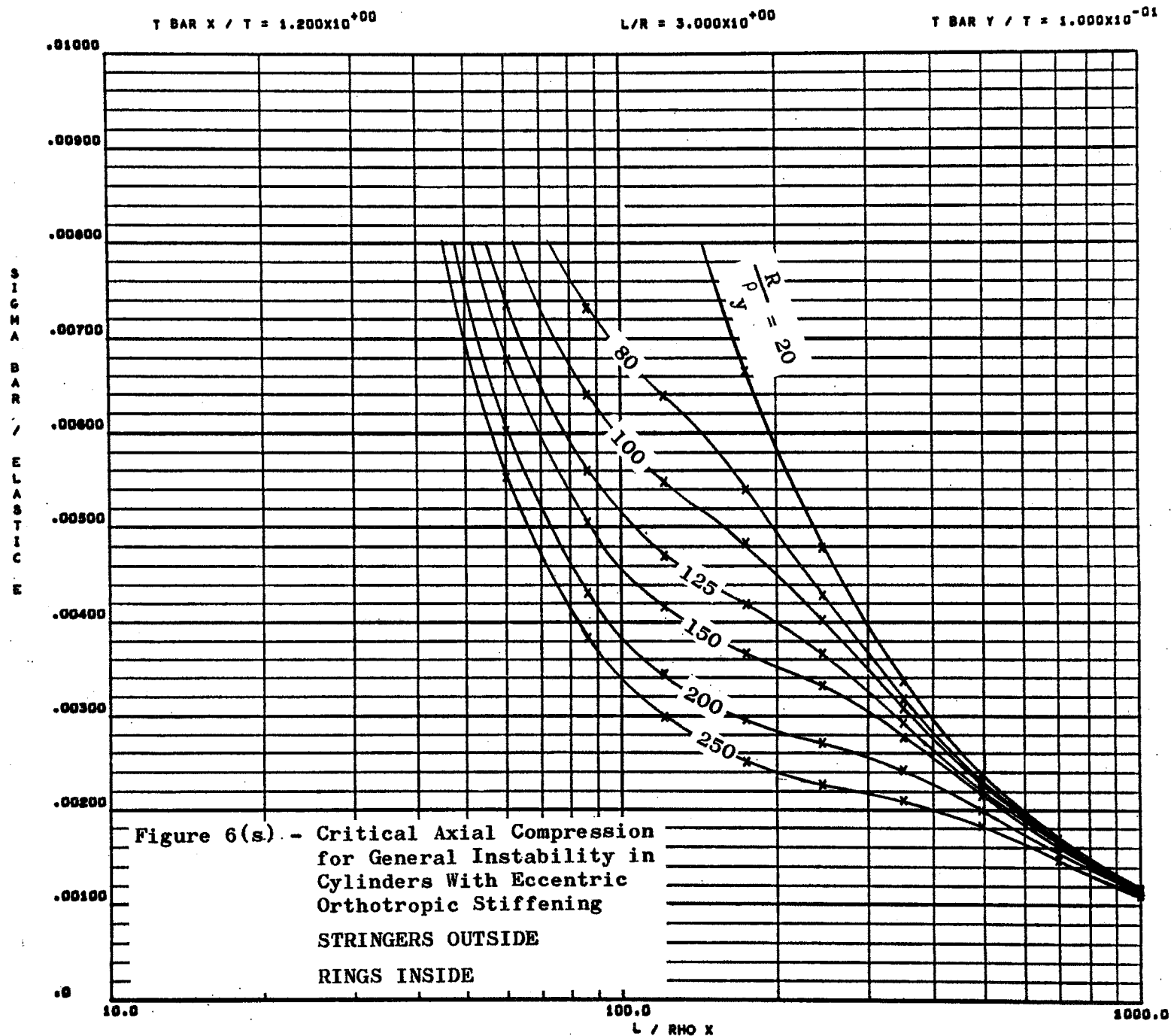


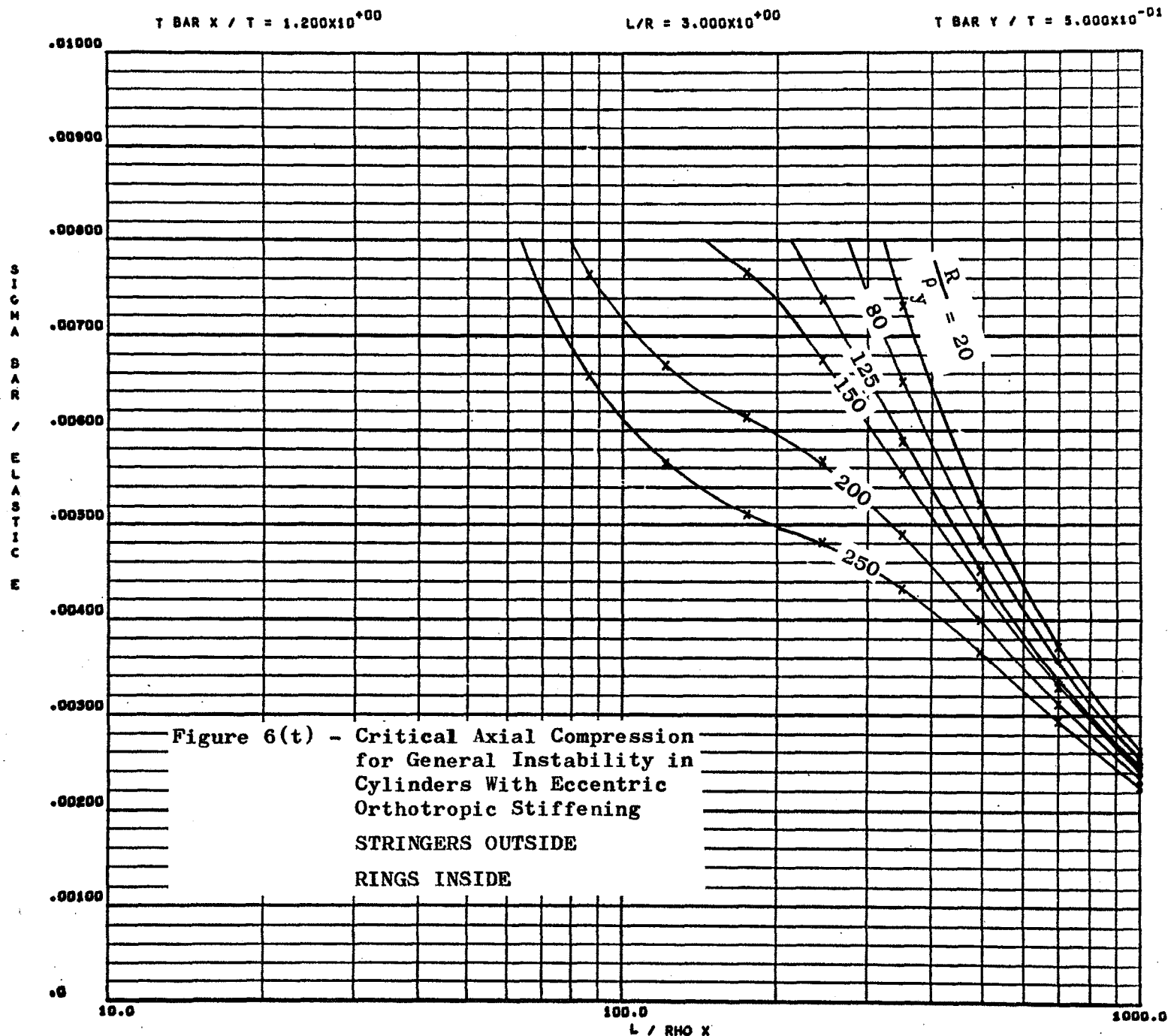


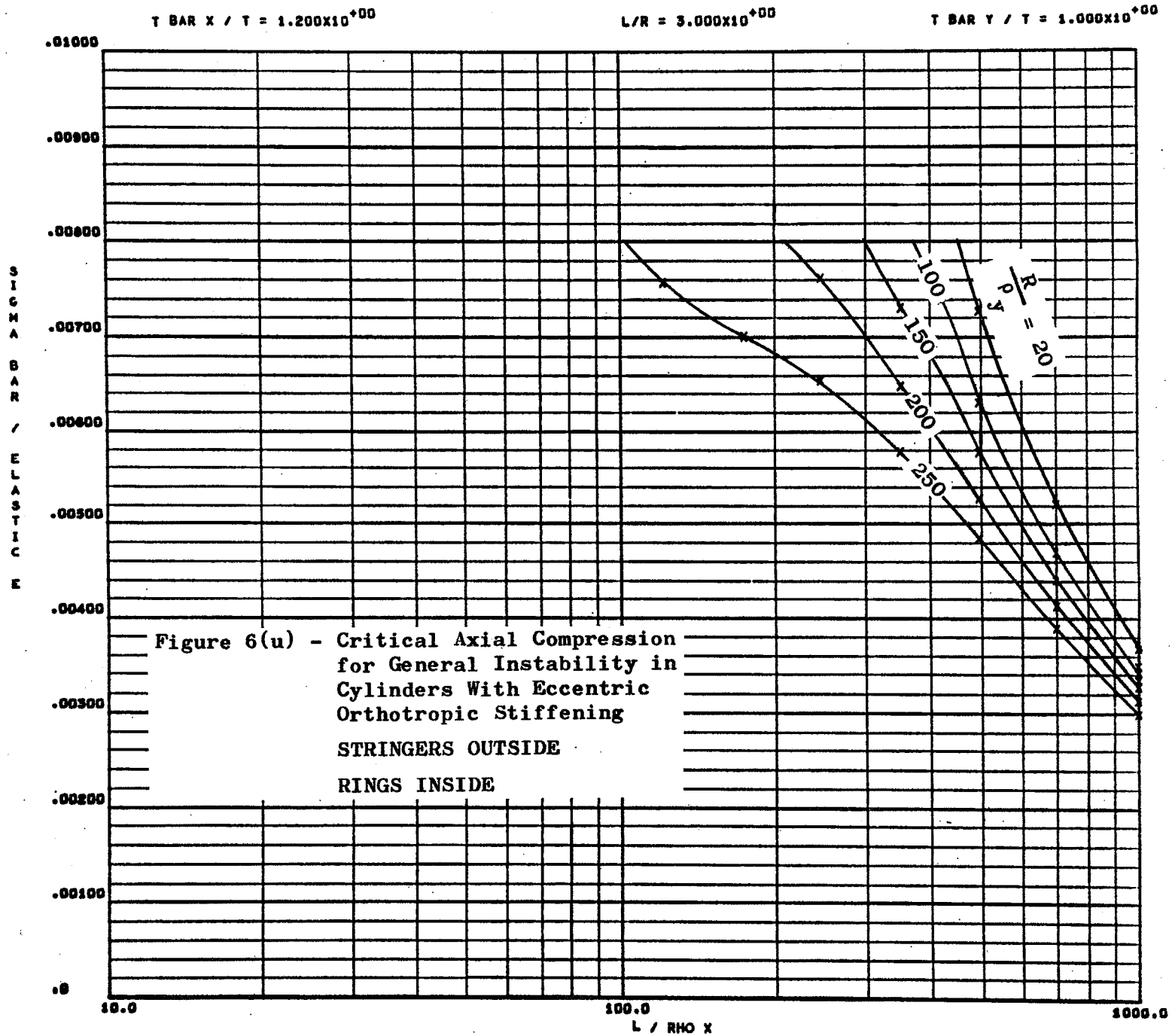


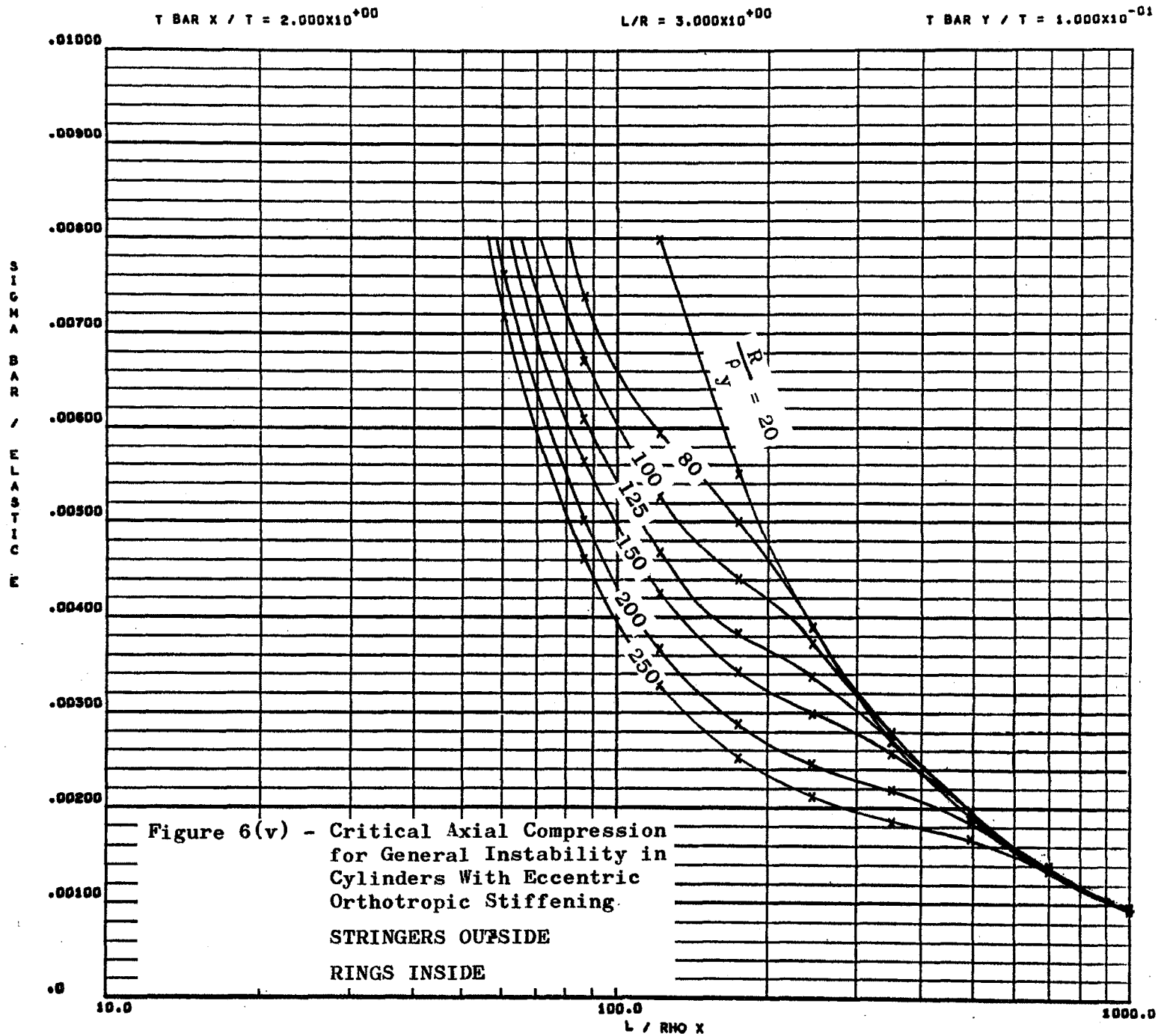


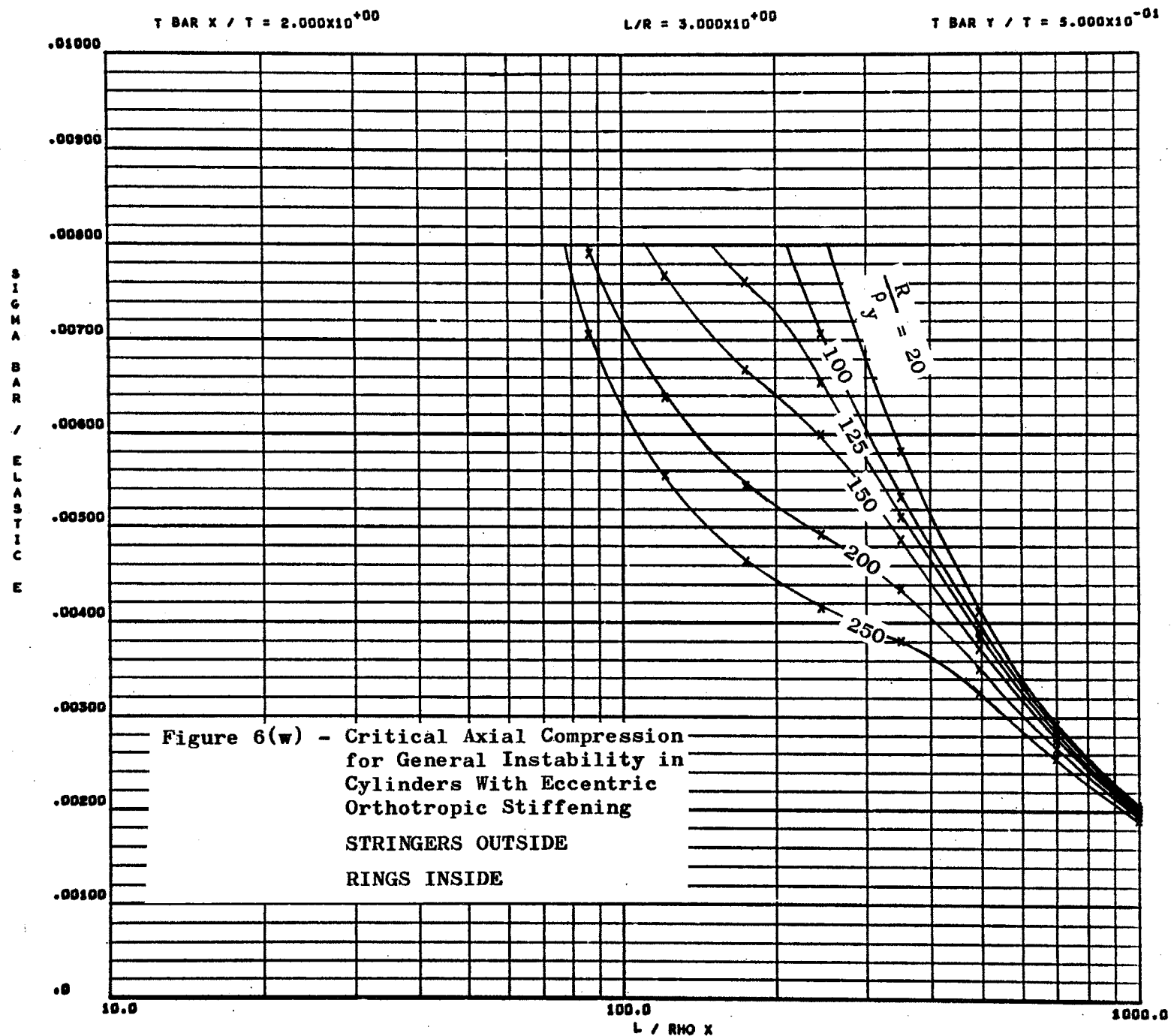


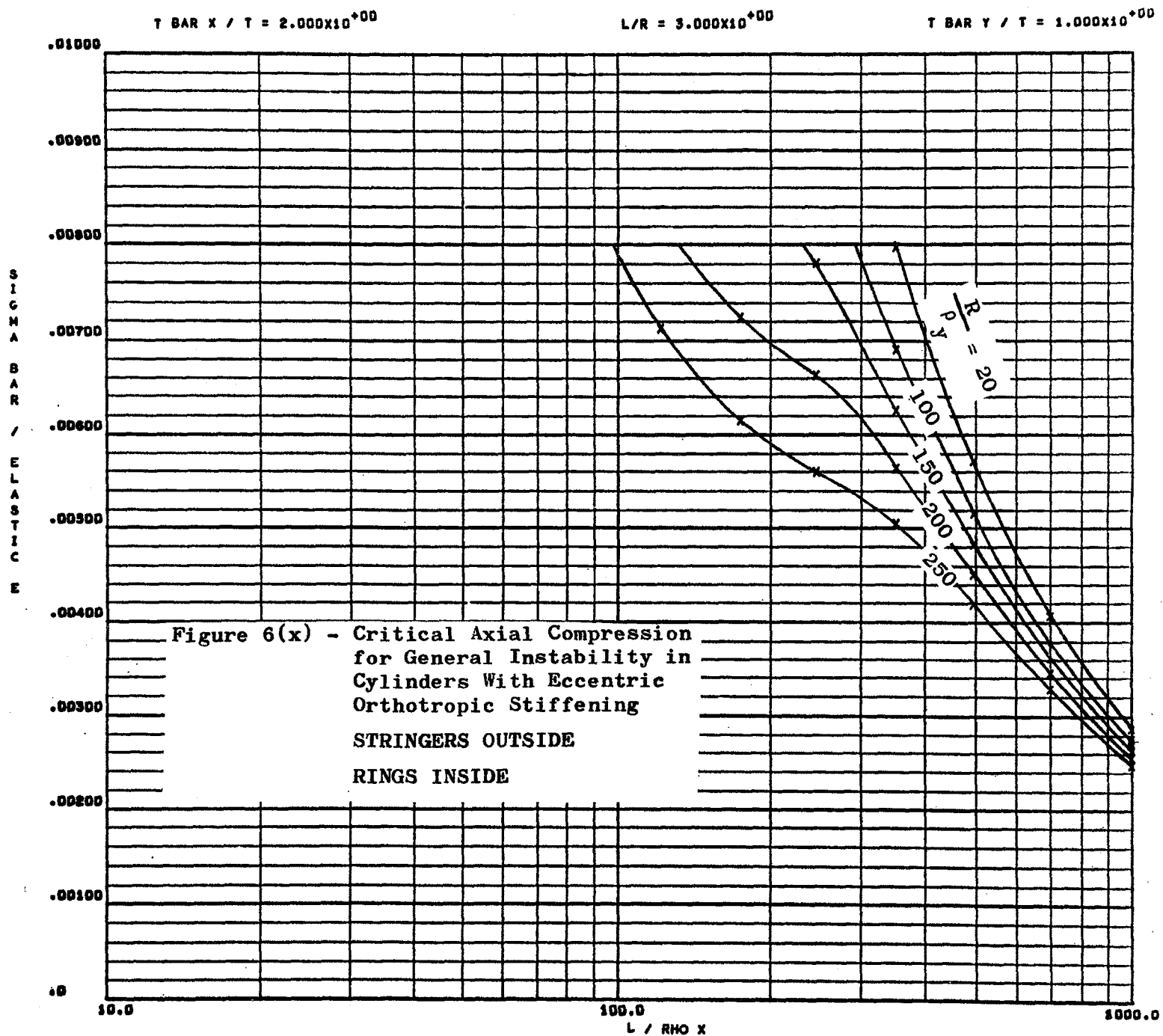


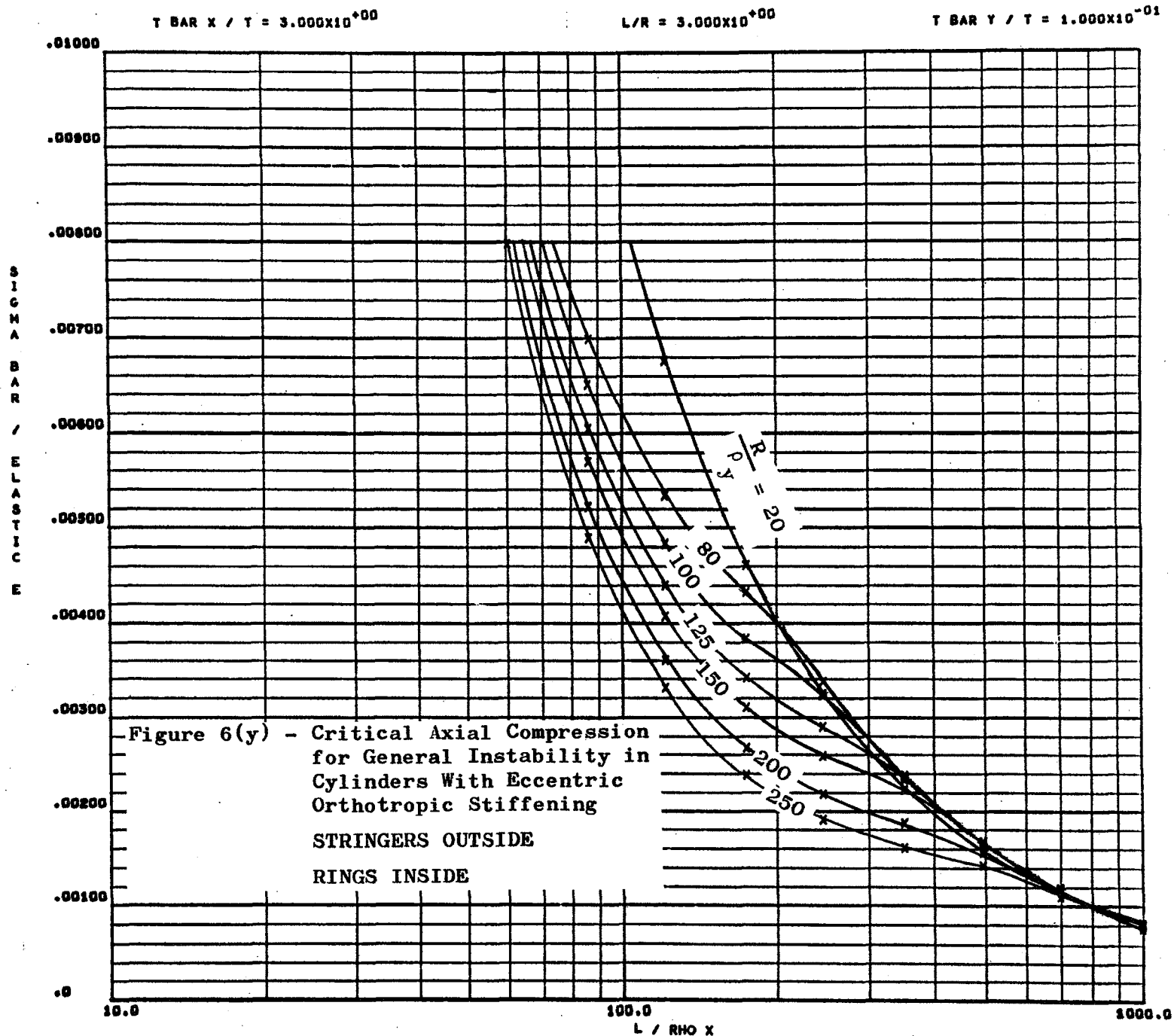


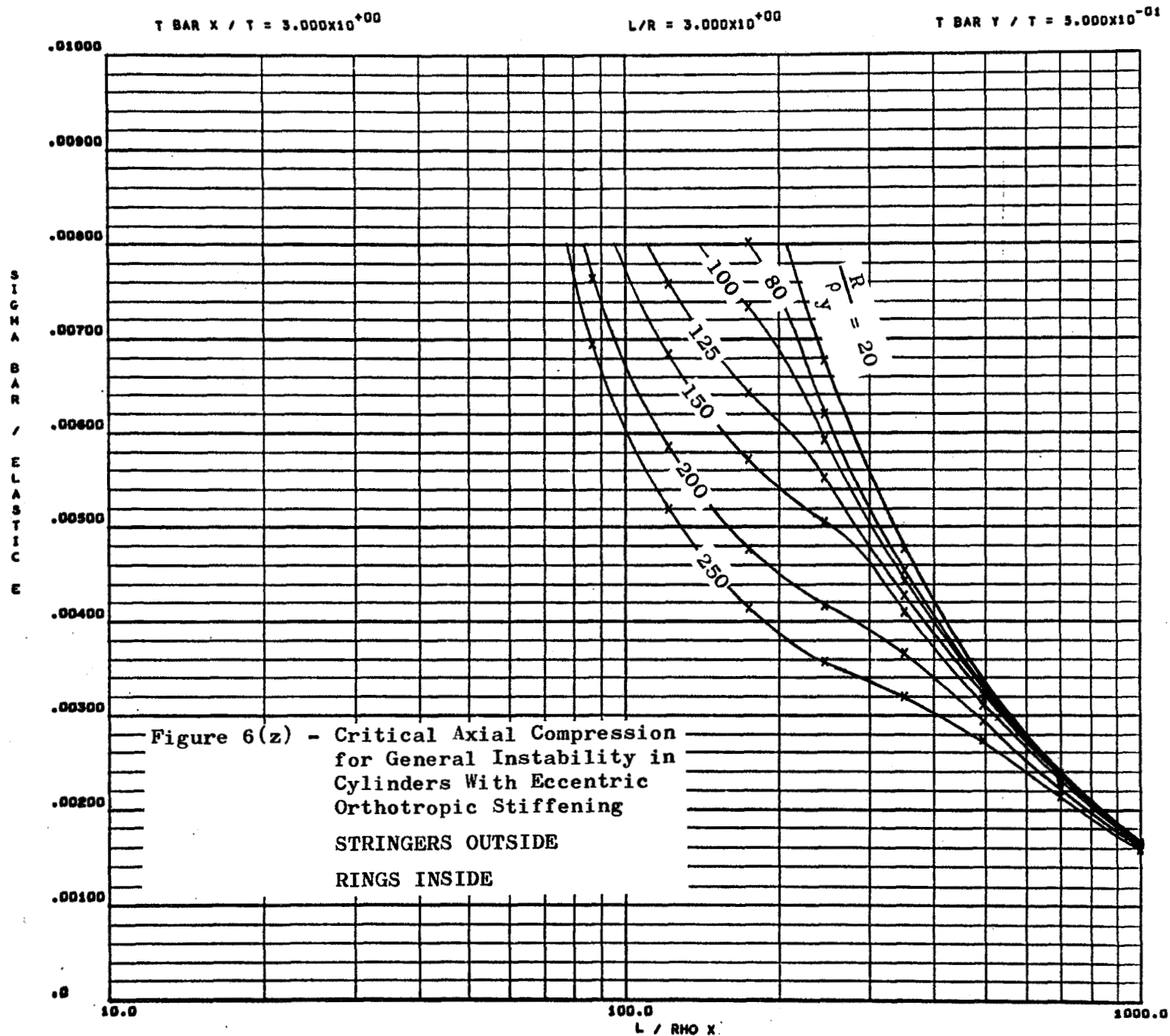


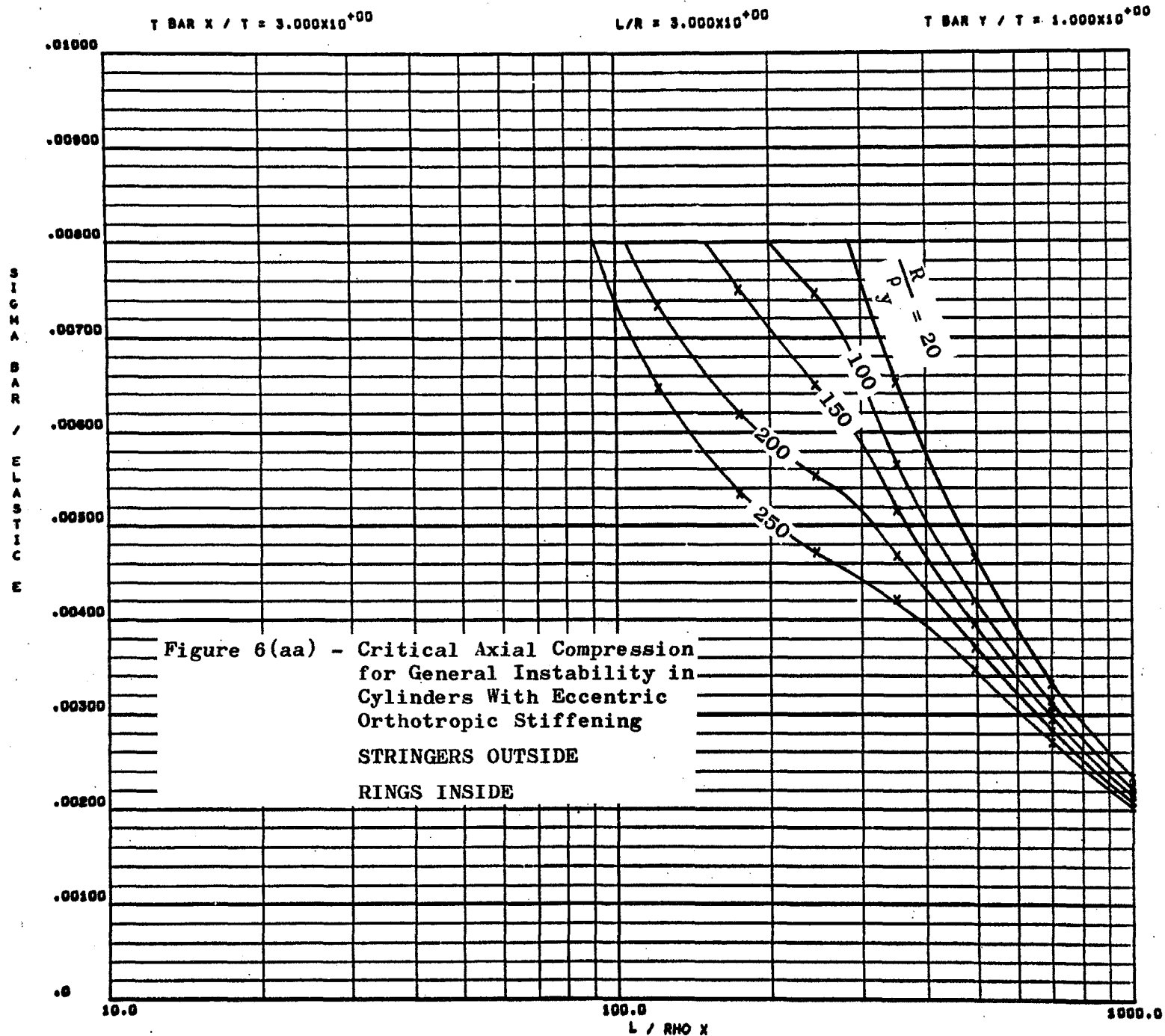


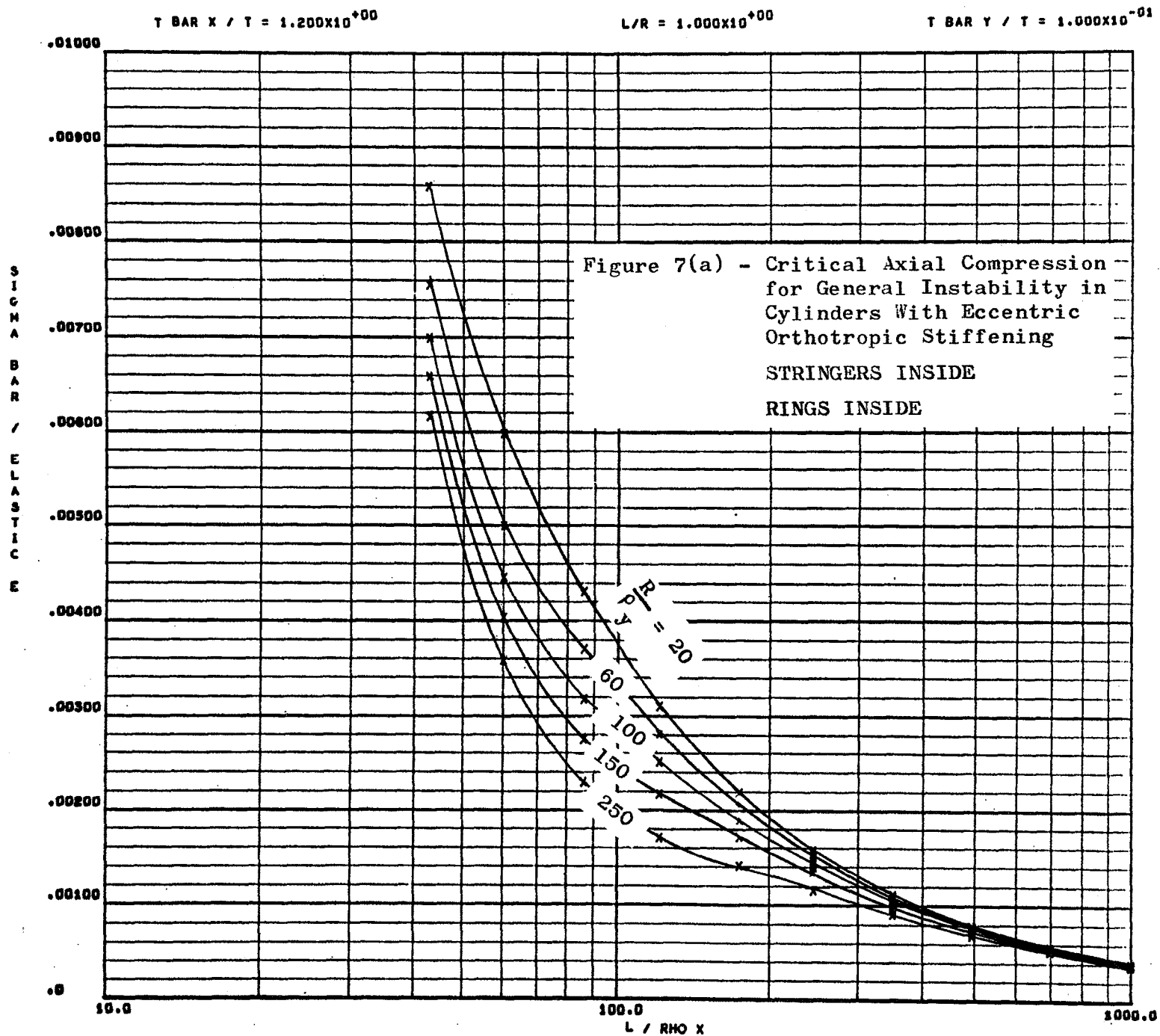


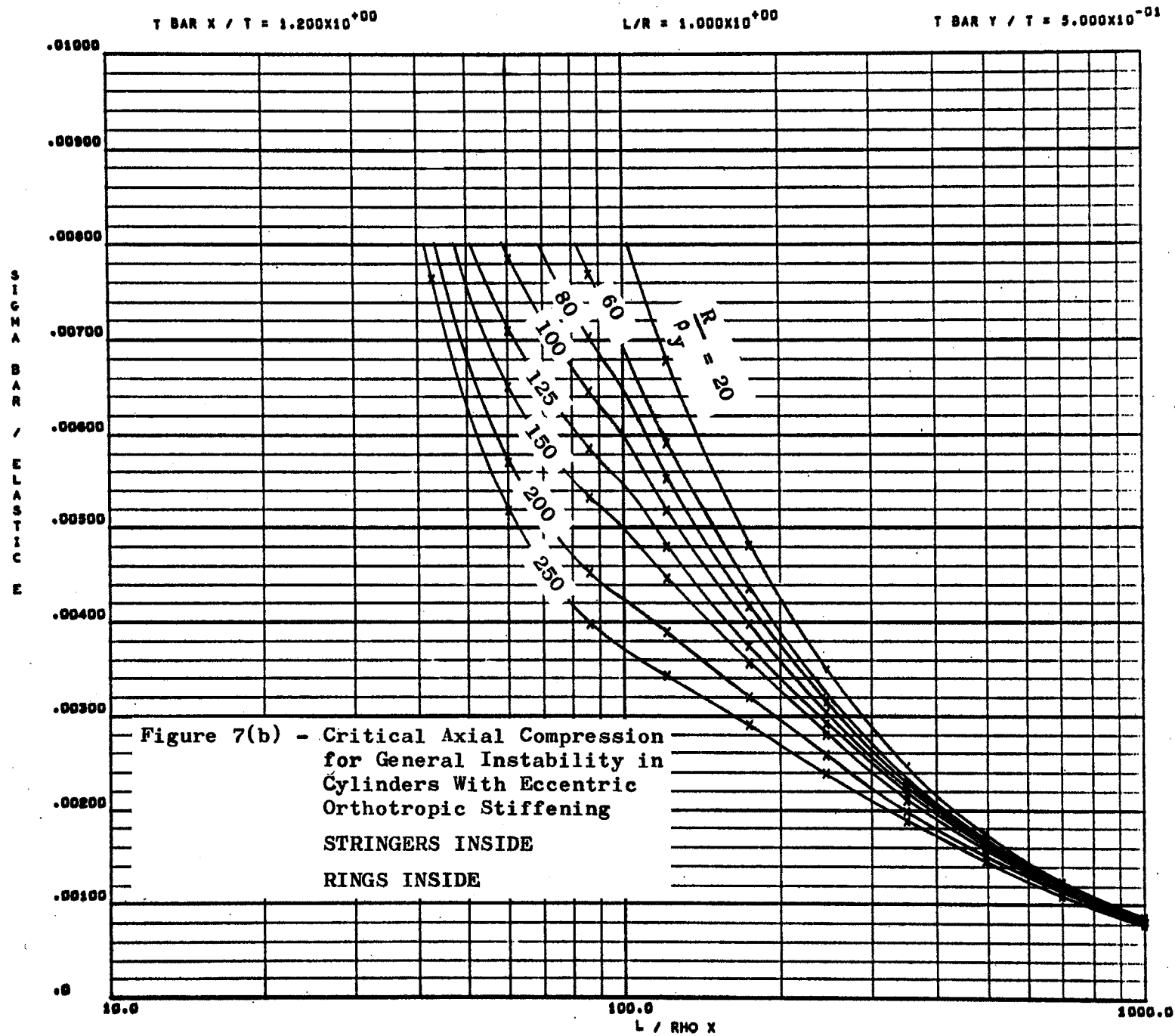


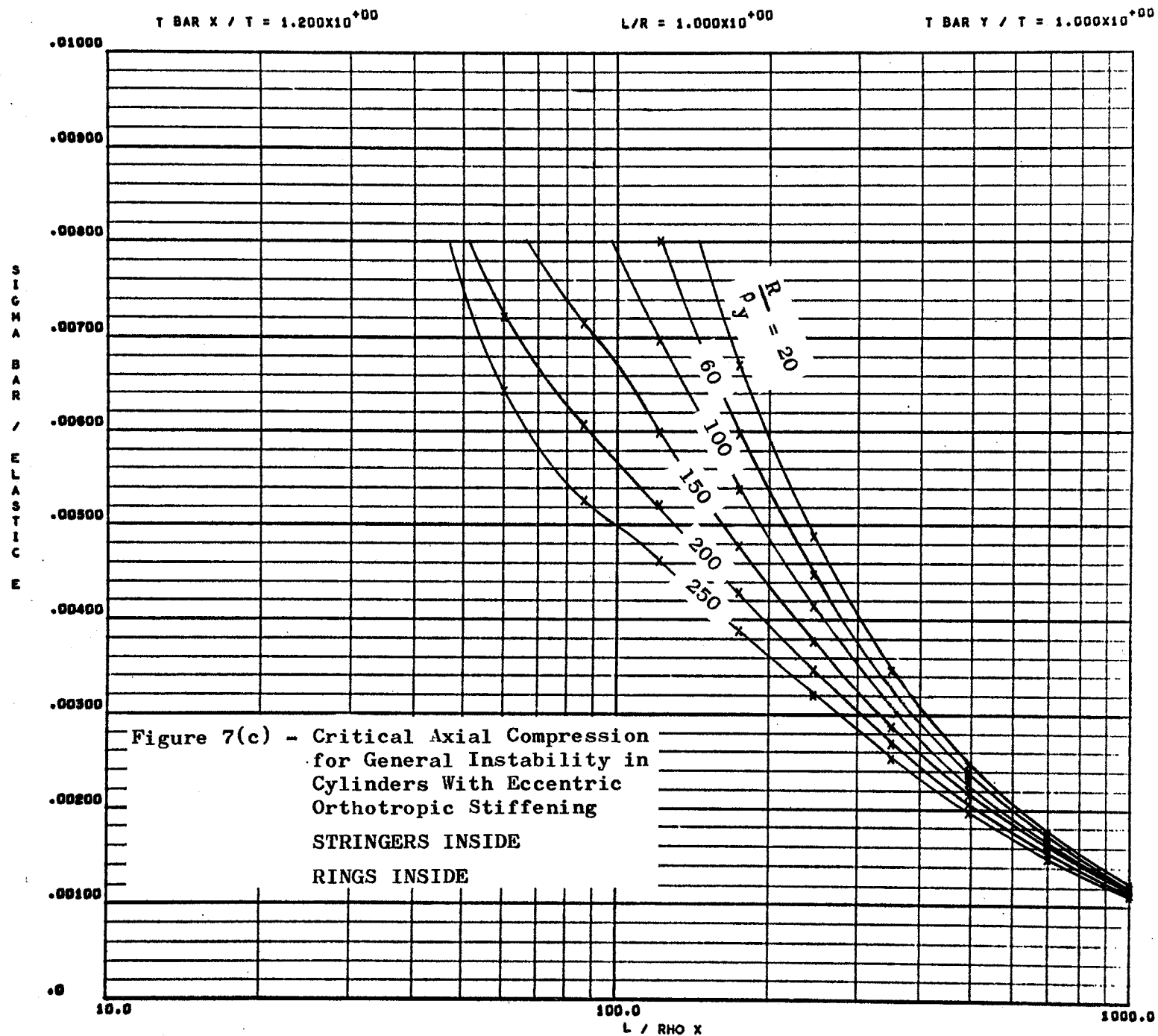


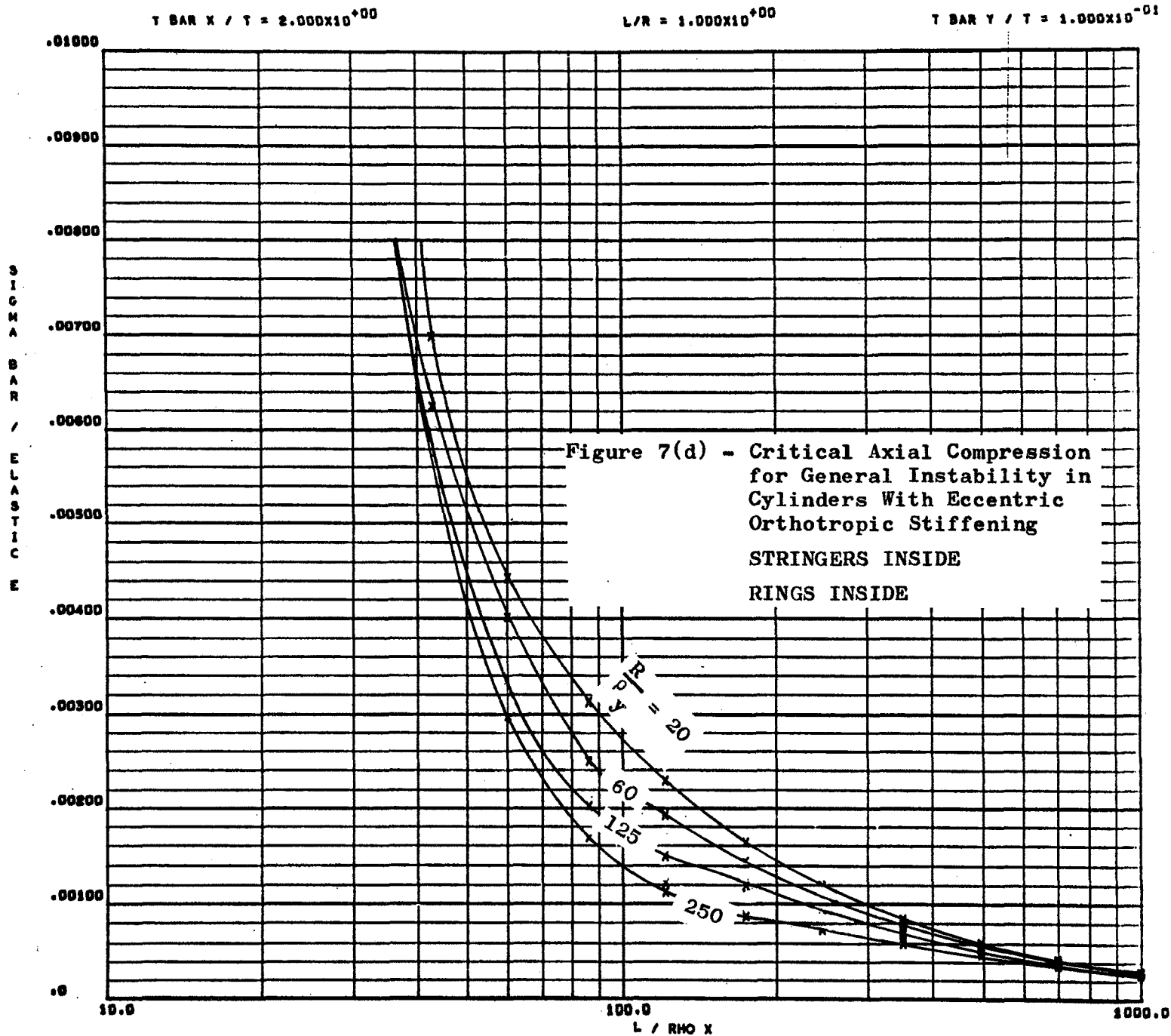


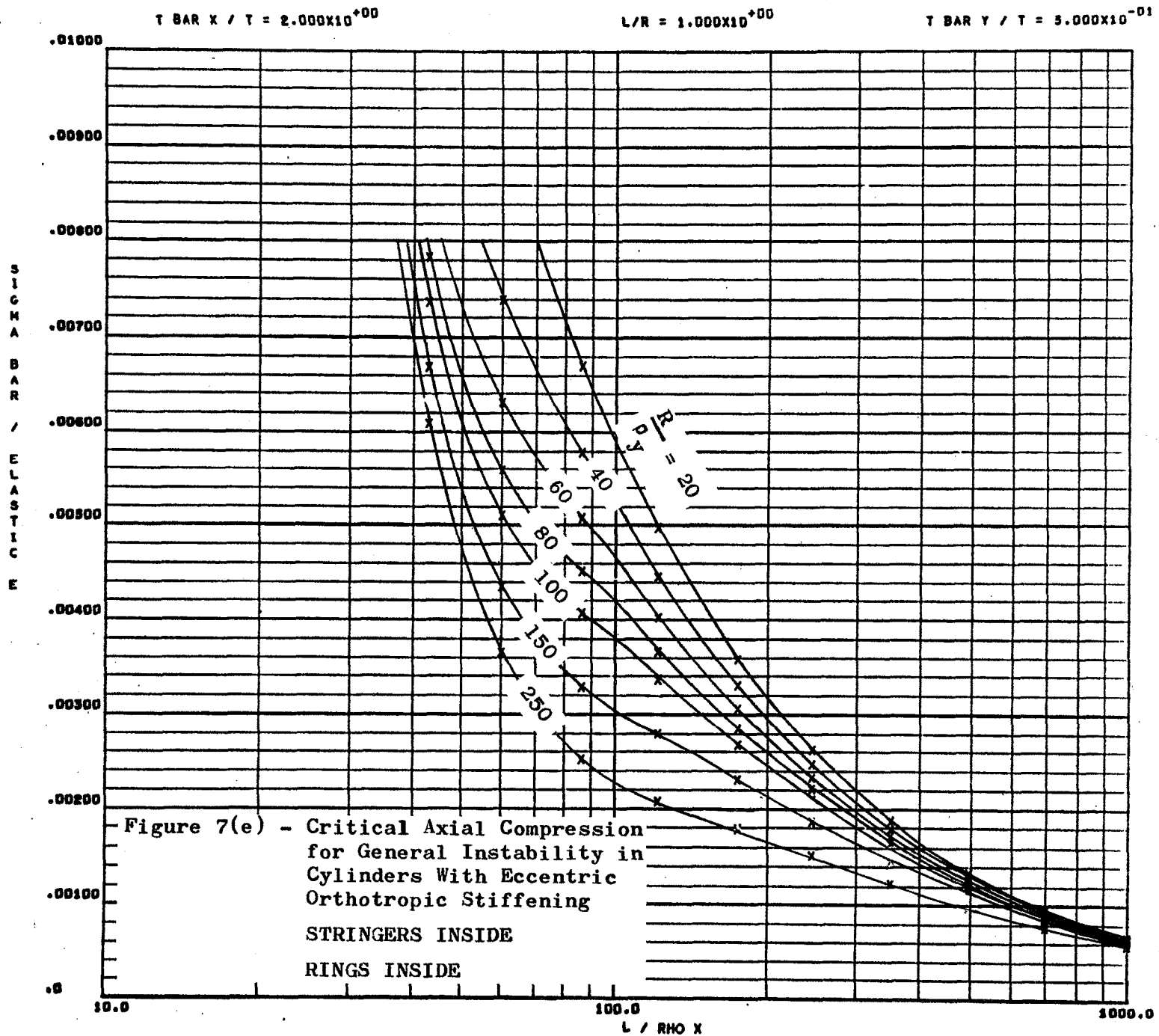


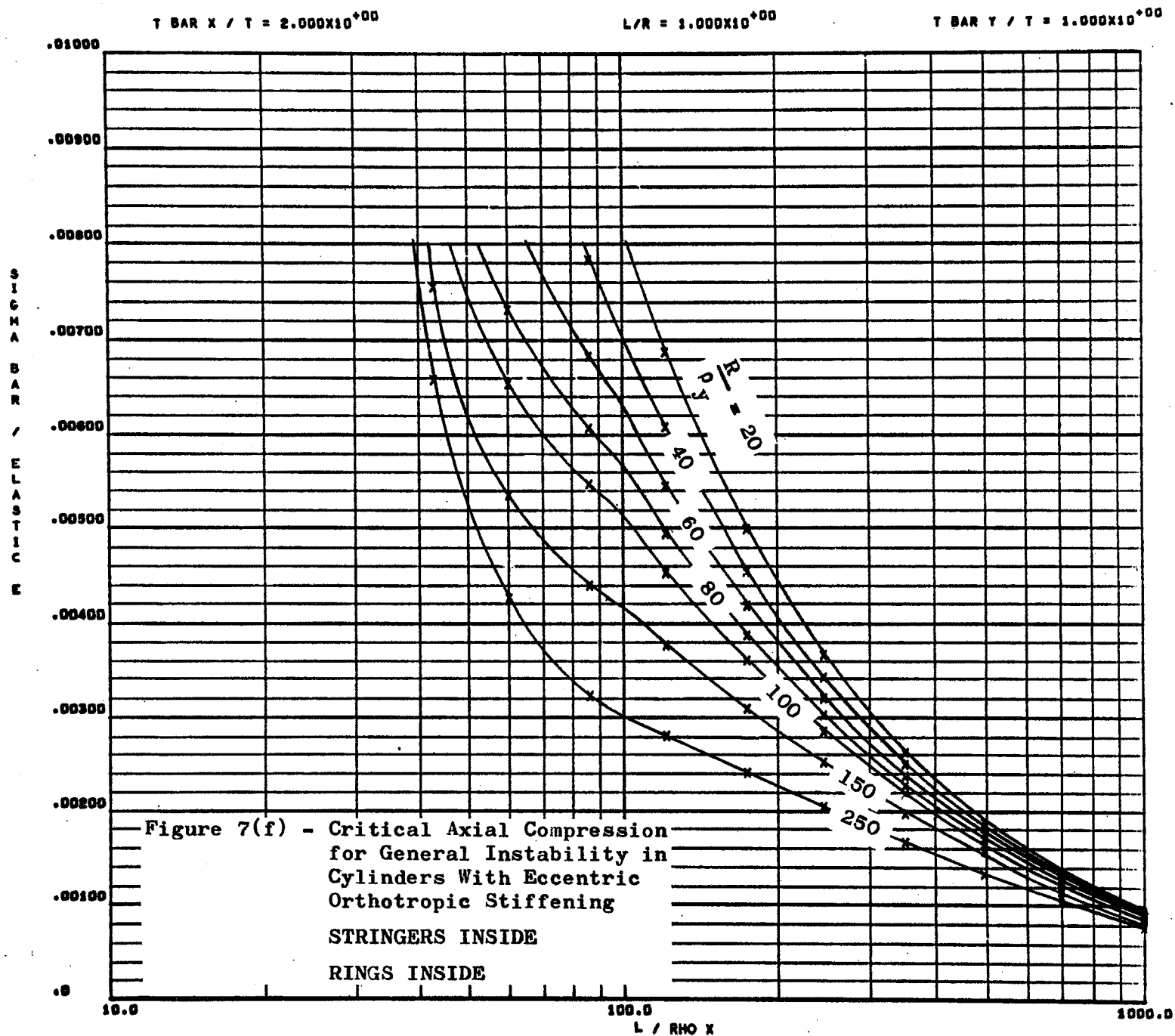


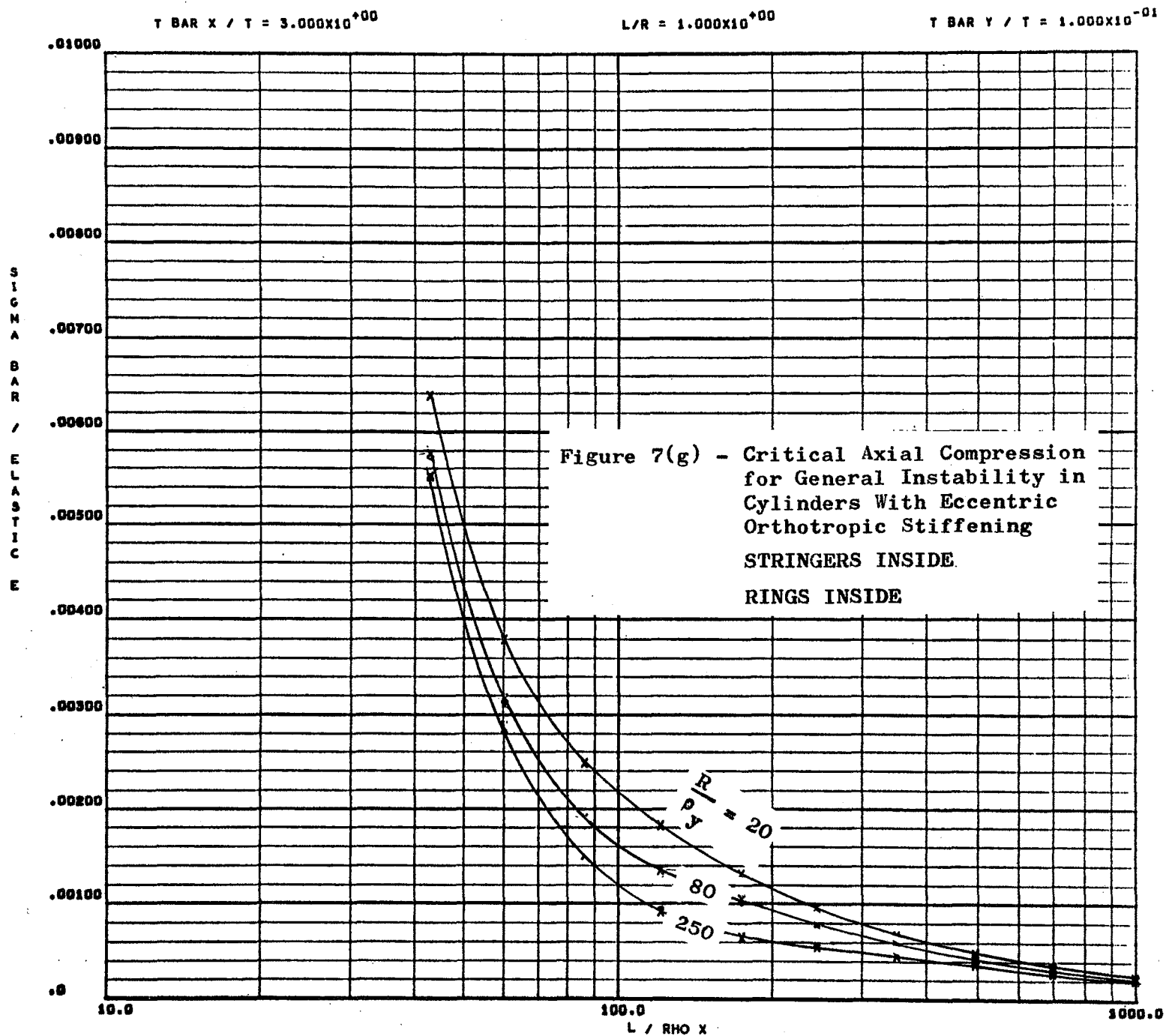








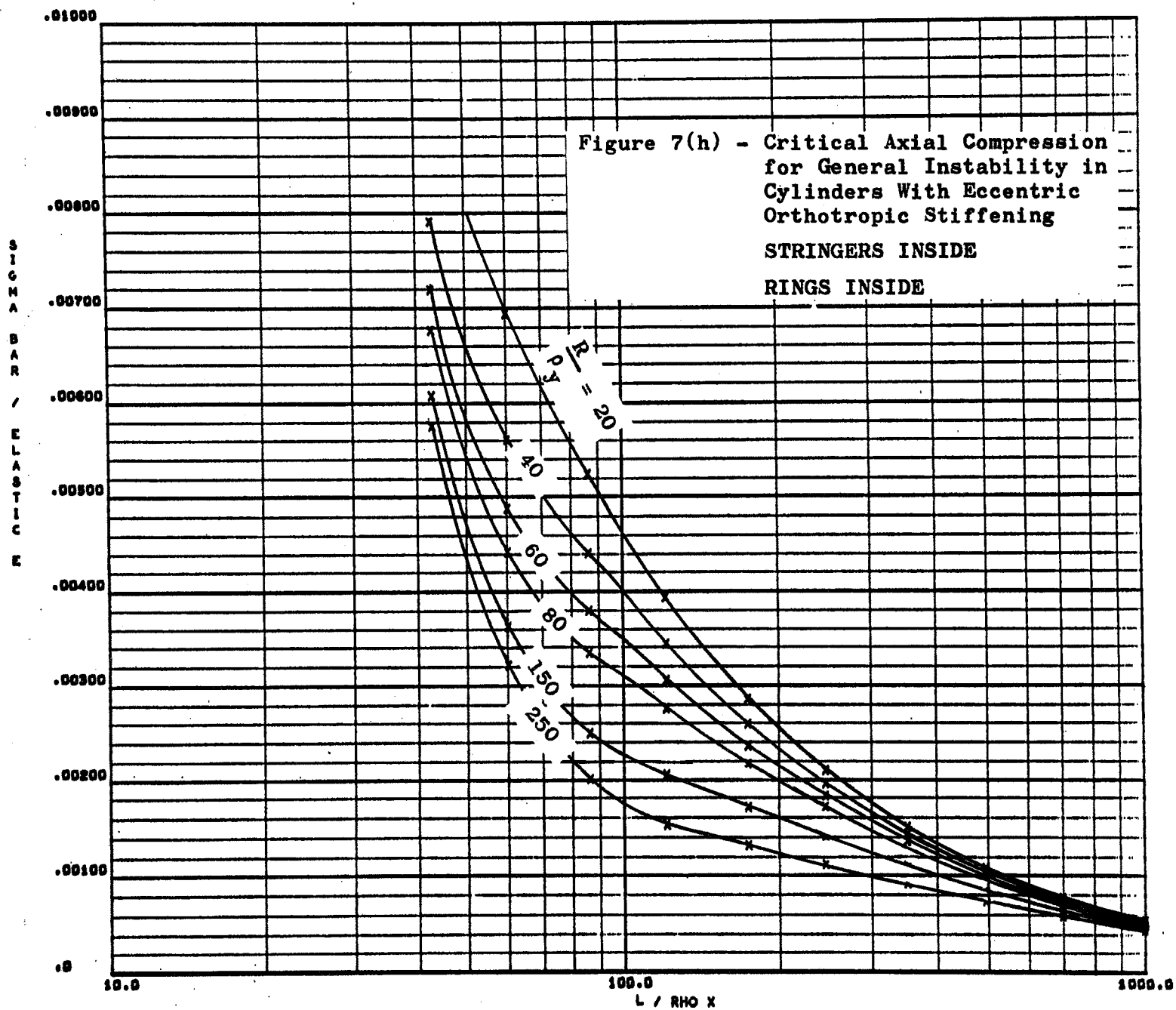




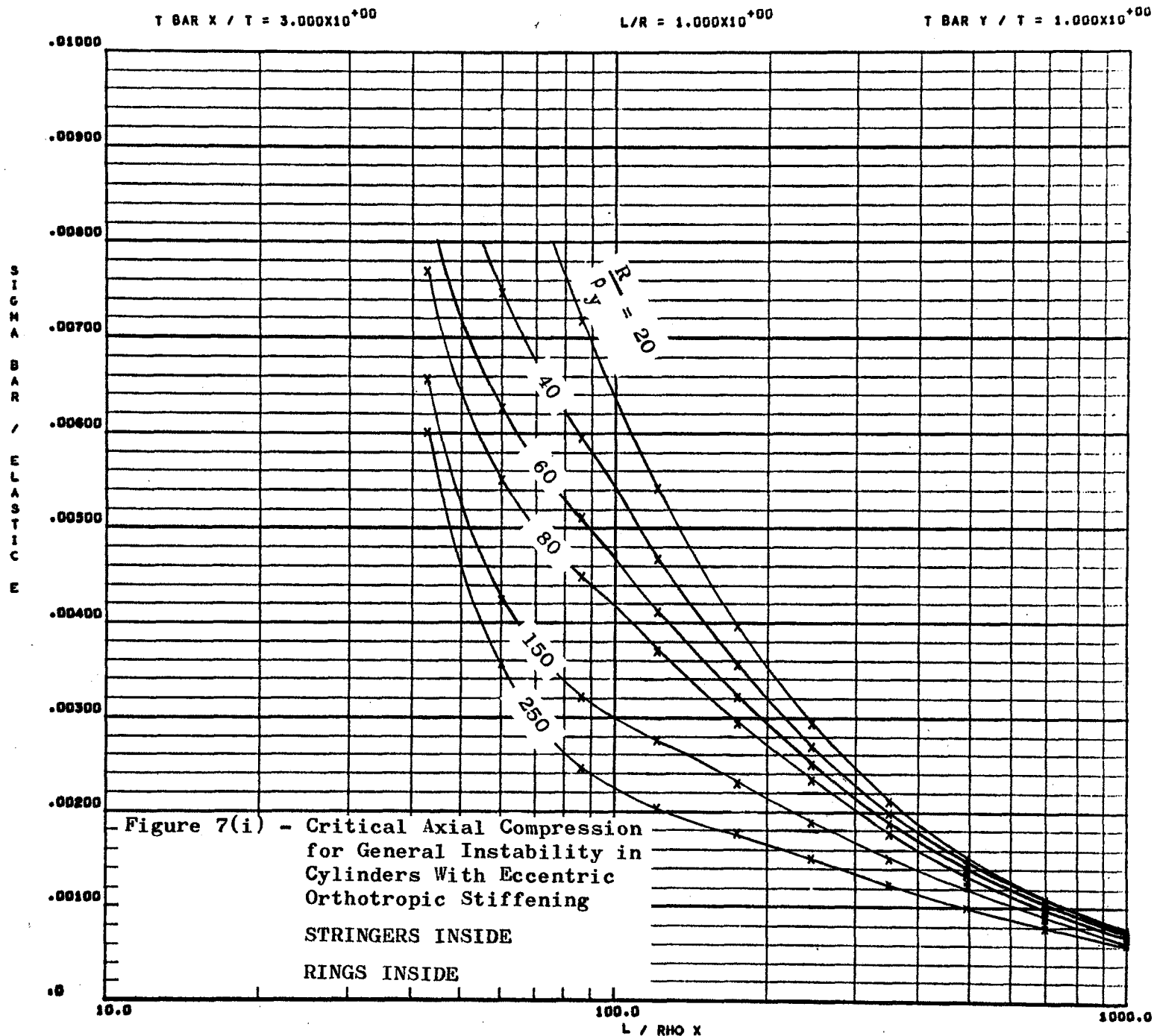
$$\bar{T}_X / T = 3.000 \times 10^{+00}$$

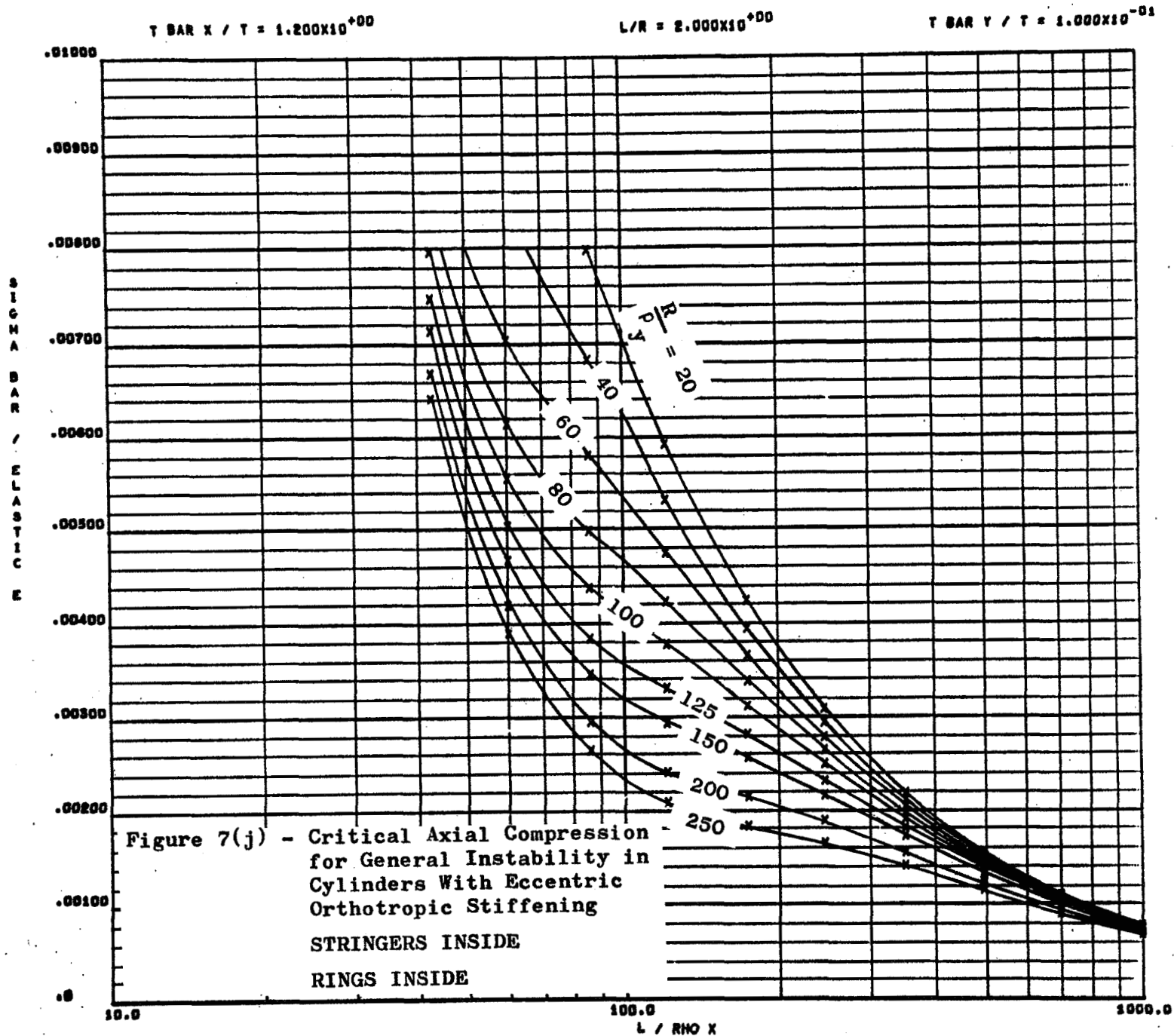
$$L/R = 1.000 \times 10^{+00}$$

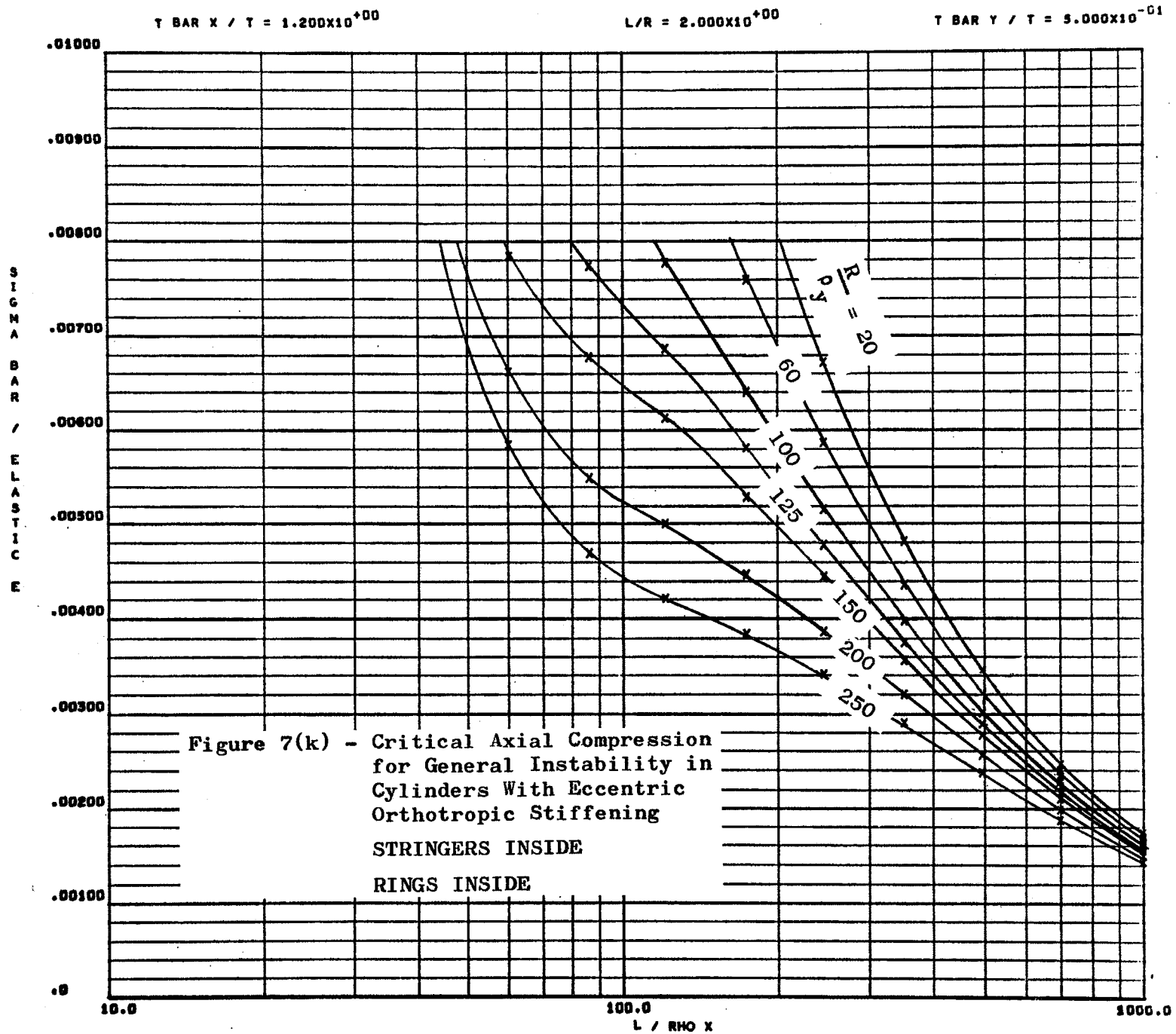
$$\bar{T}_Y / T = 9.000 \times 10^{-01}$$

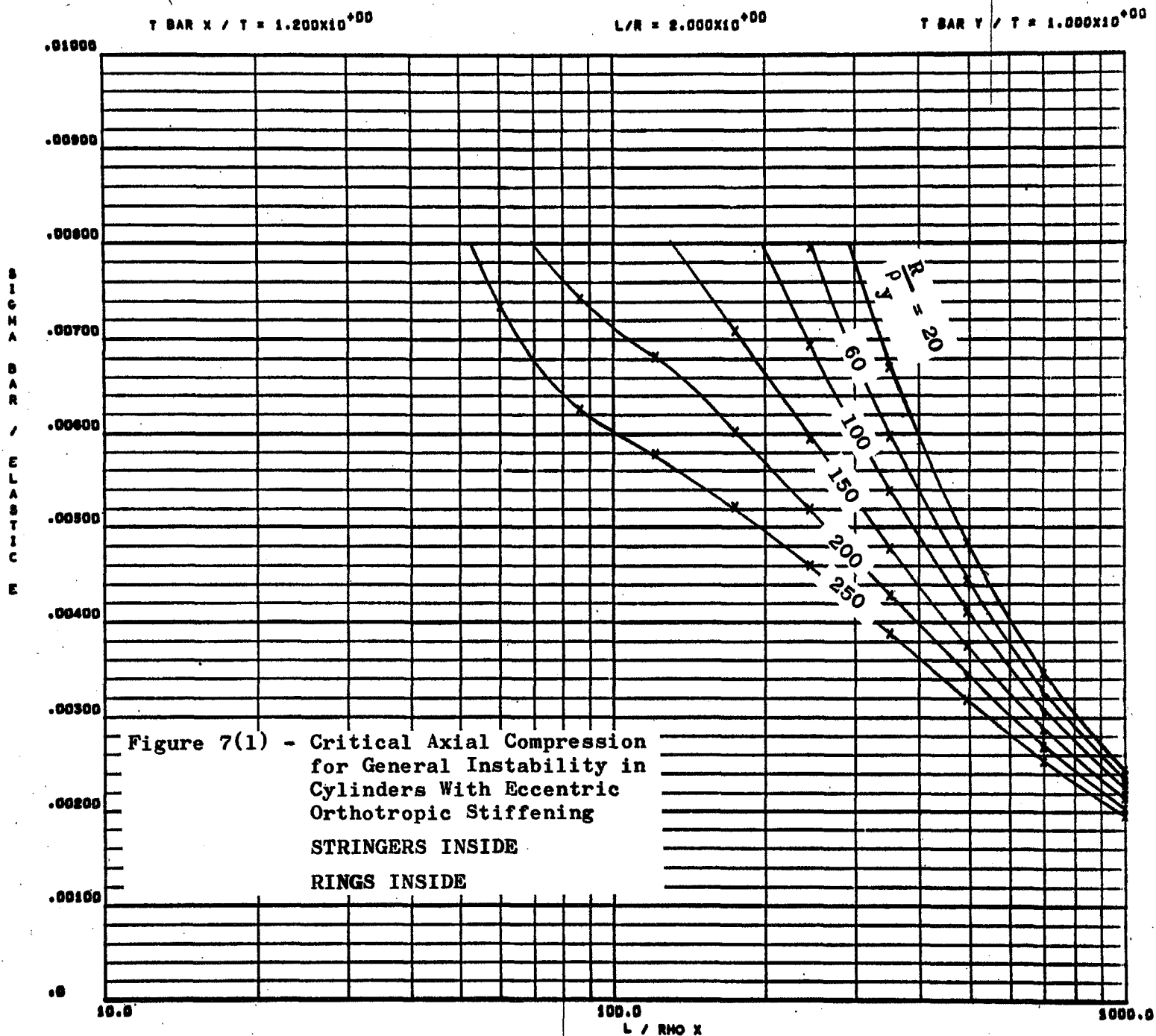


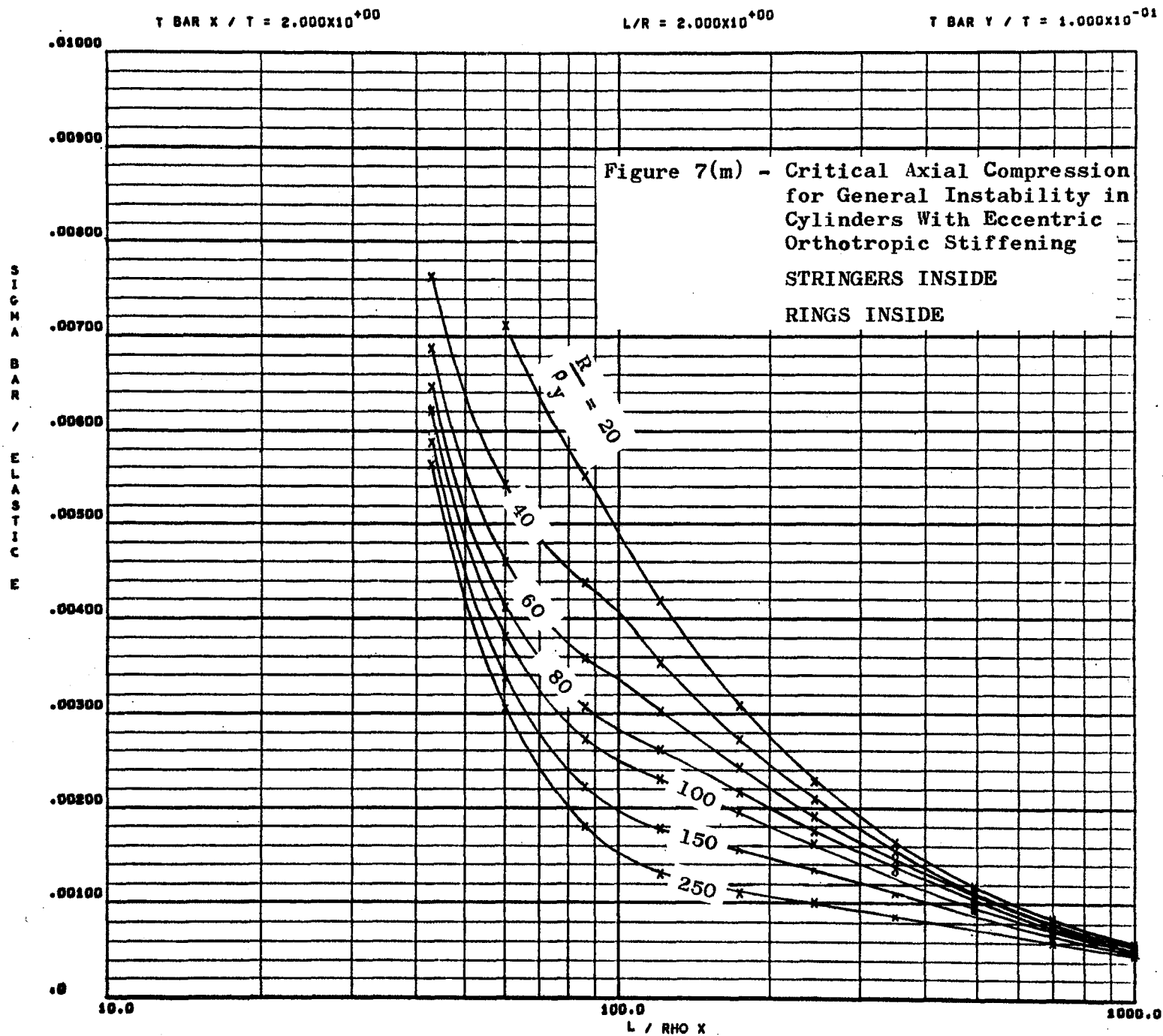
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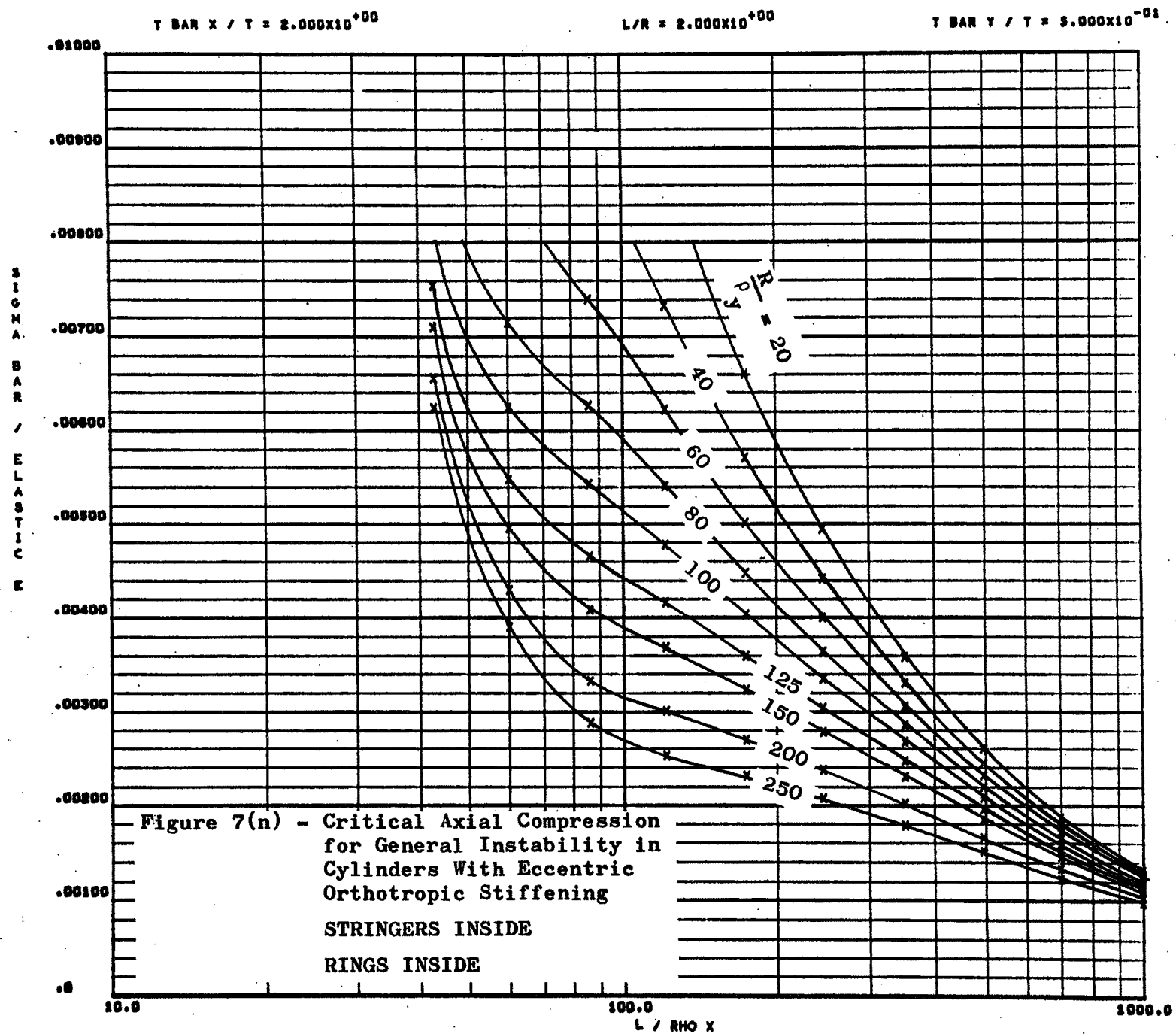


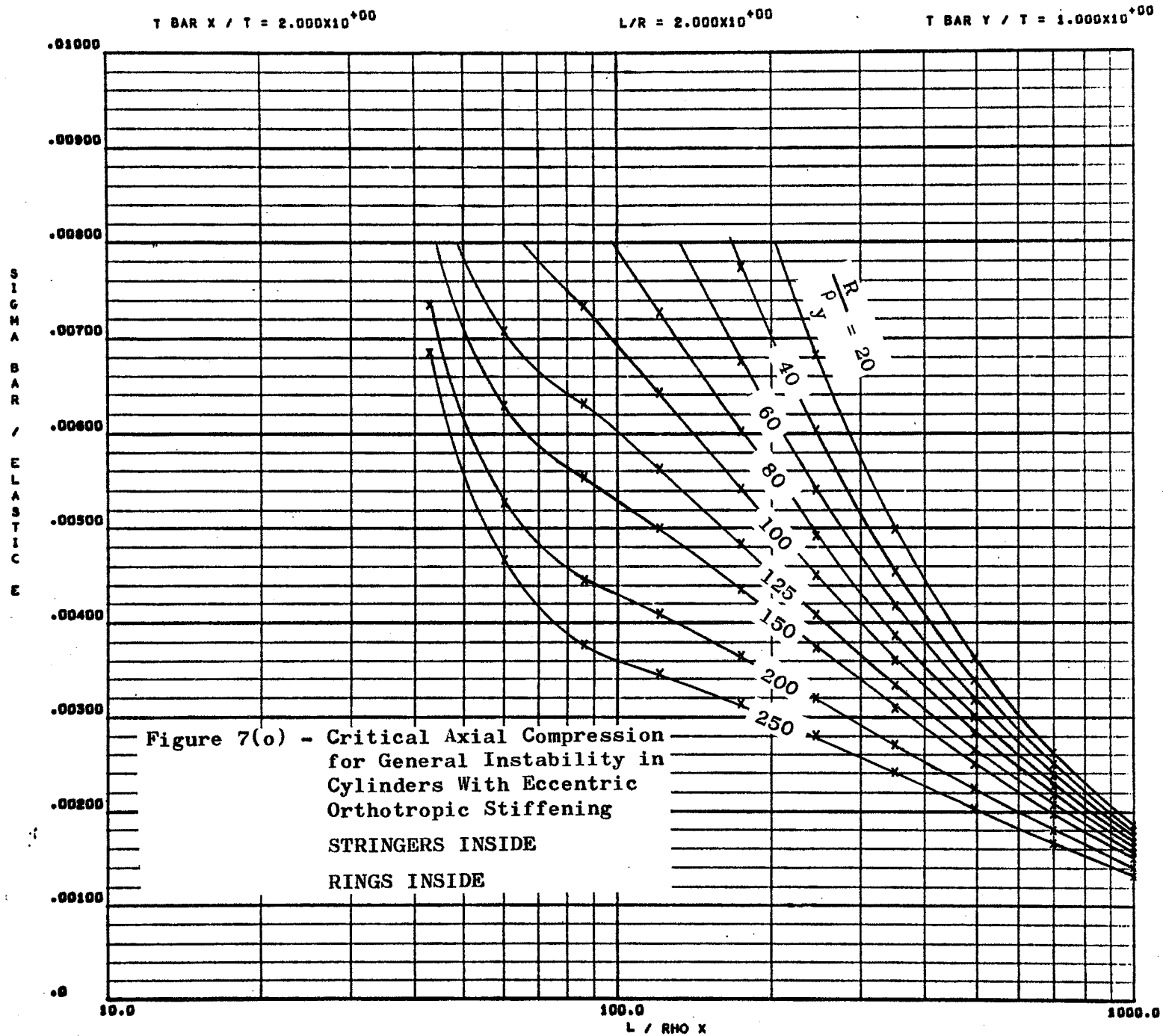








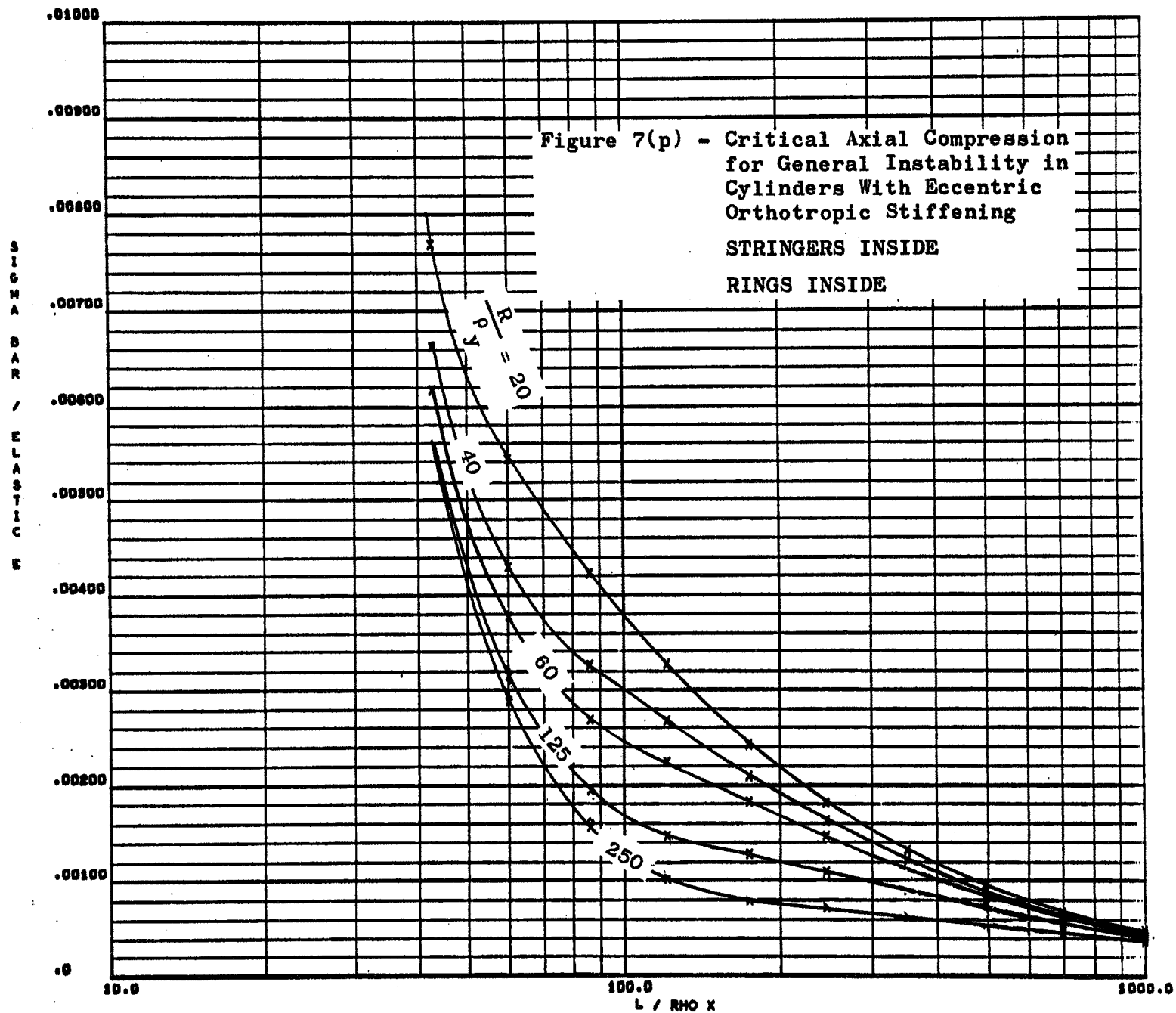




$$\bar{T}_X / T = 3.000 \times 10^{-00}$$

$$L/R = 2.000 \times 10^{-00}$$

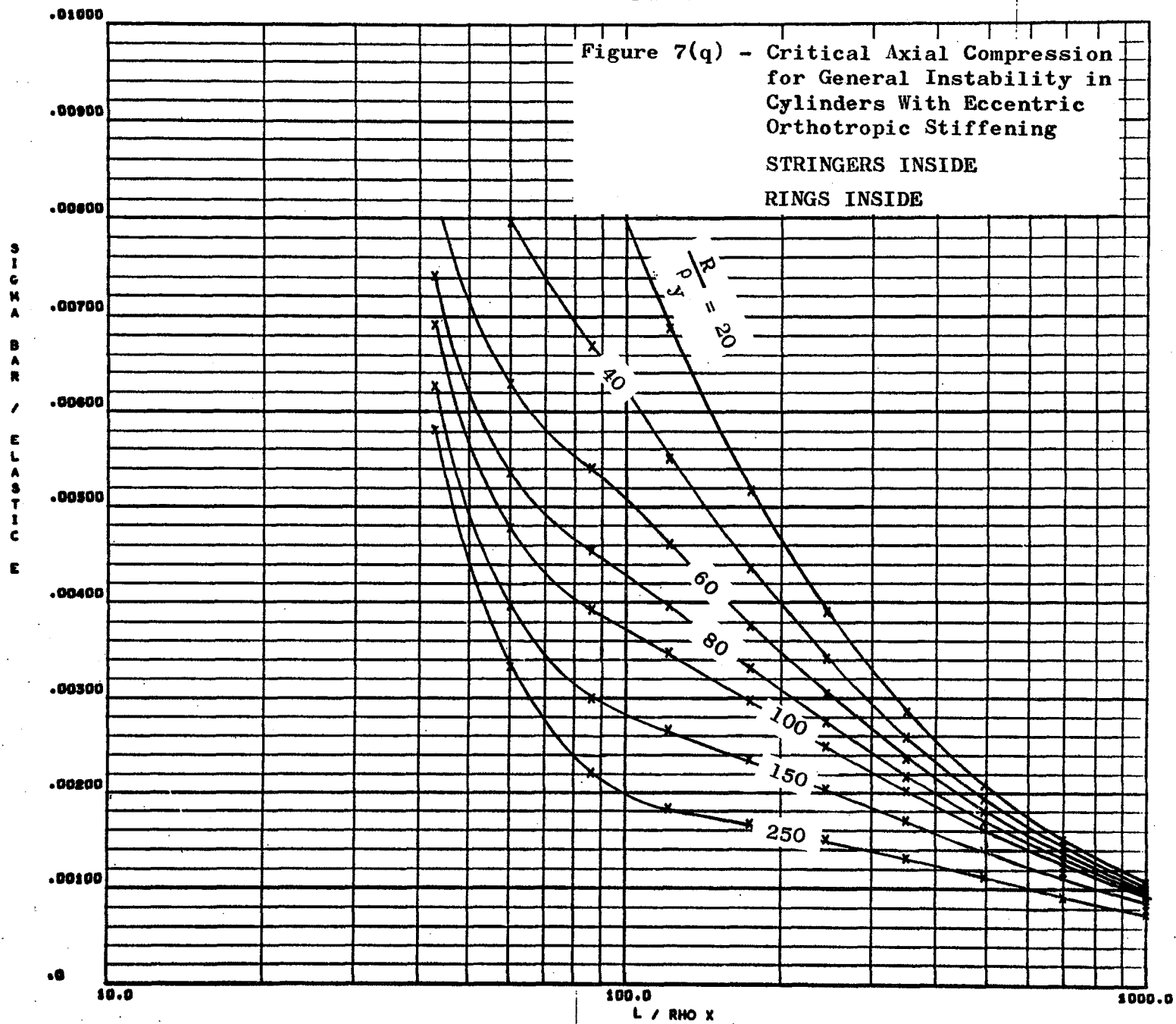
$$\bar{T}_Y / T = 1.000 \times 10^{-01}$$



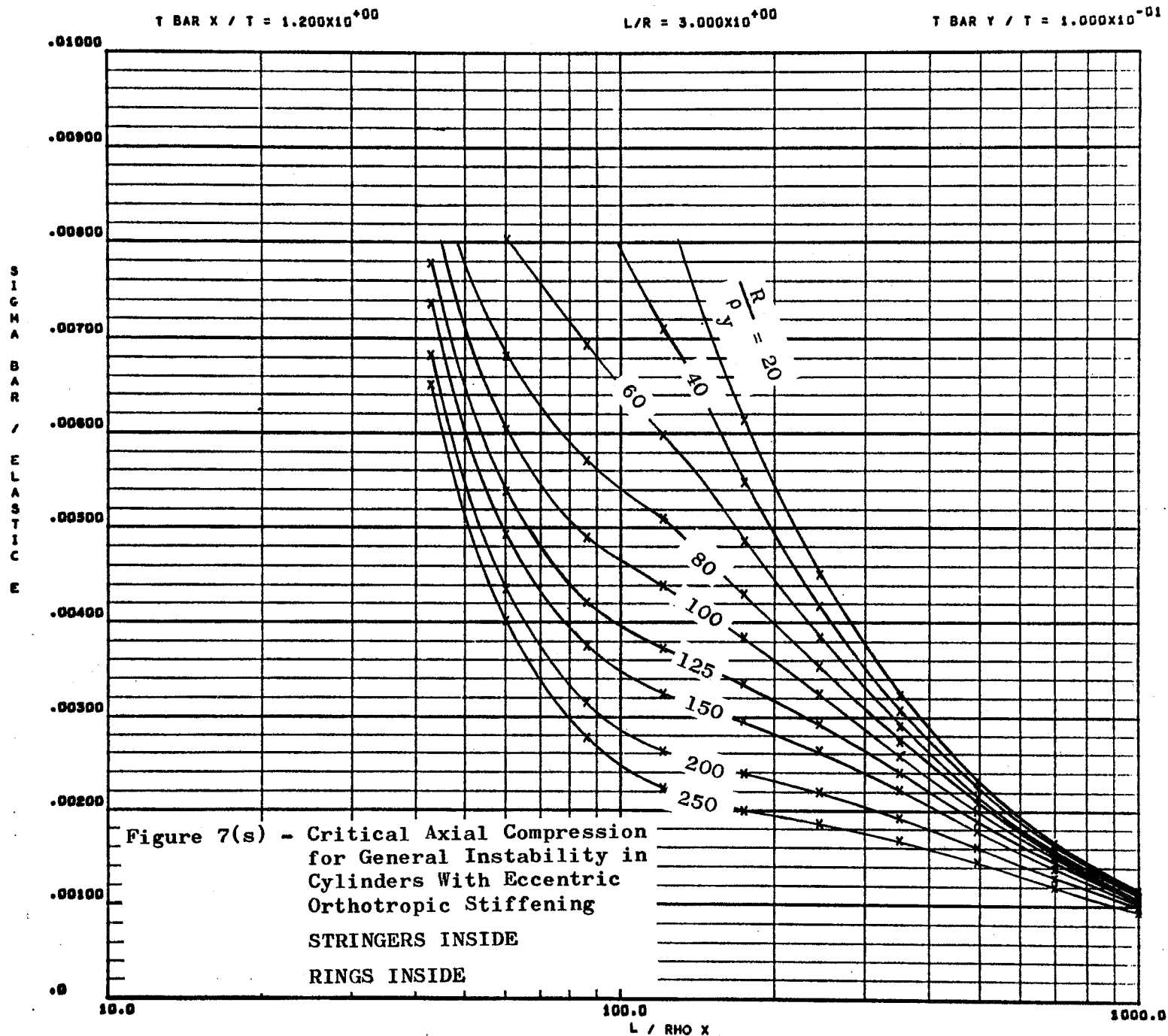
$$\bar{T}_X / T = 3.000 \times 10^{+00}$$

$$L/R = 2.000 \times 10^{+00}$$

$$\bar{T}_Y / T = 5.000 \times 10^{-01}$$



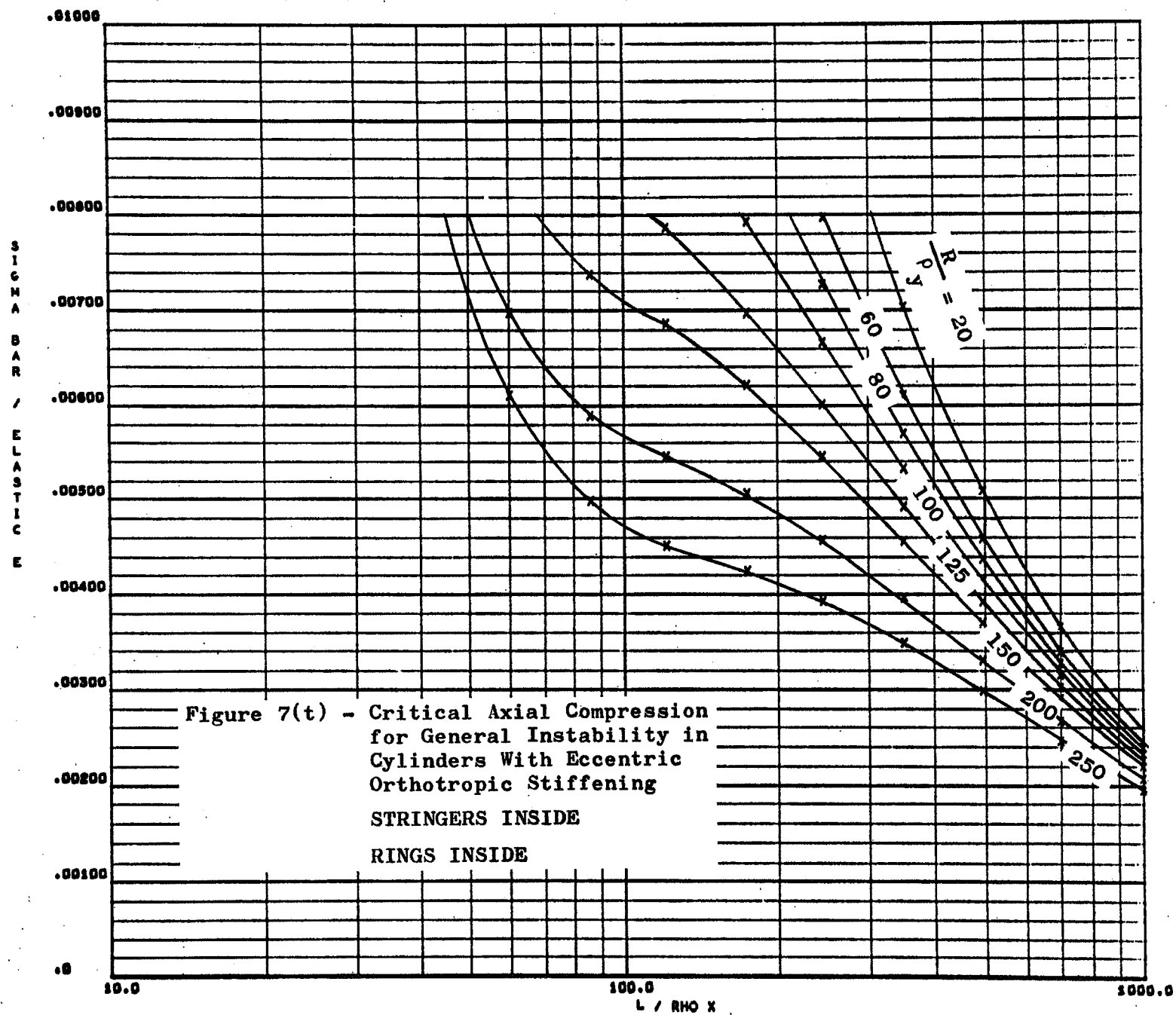


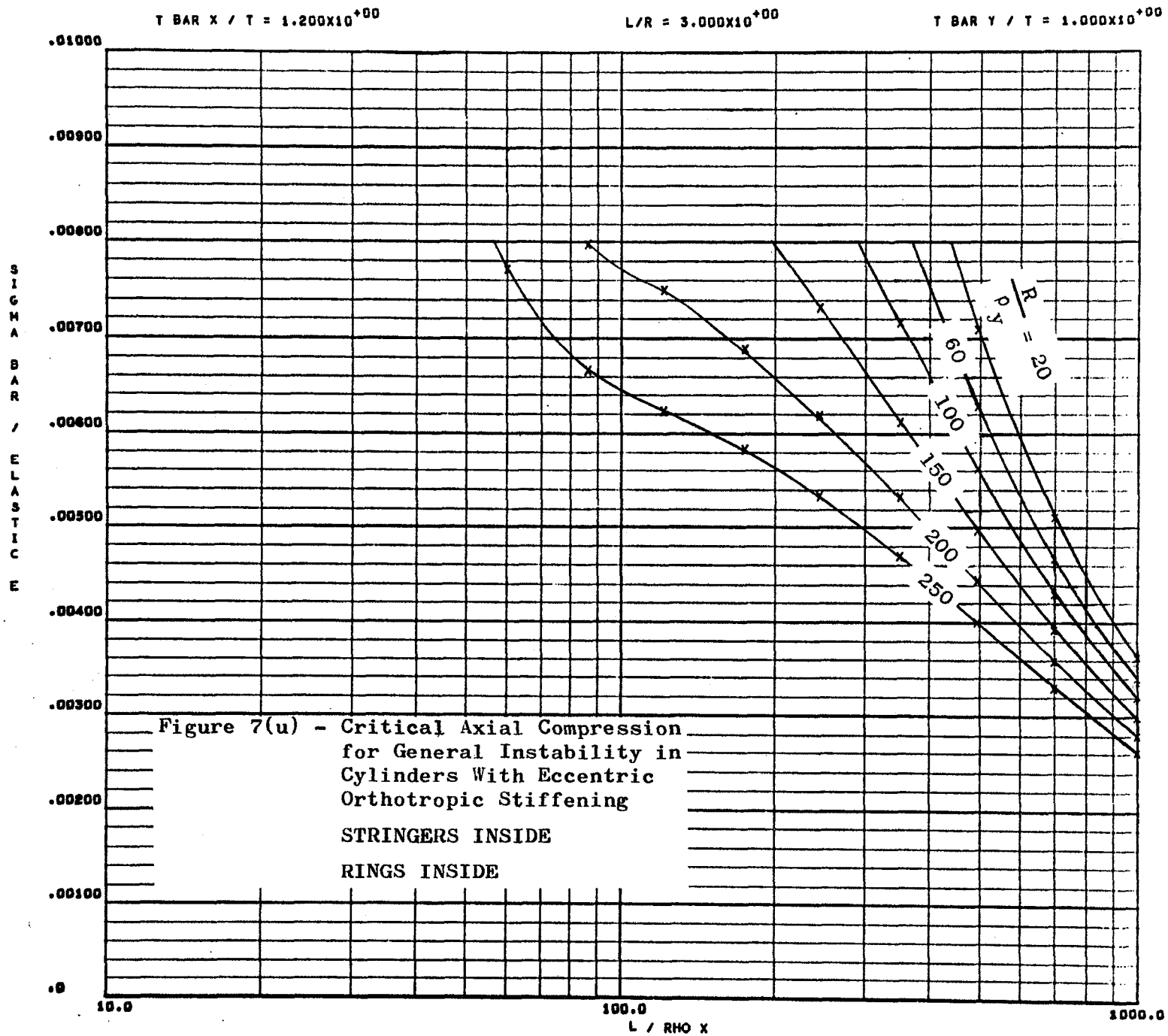


$$\bar{T}_X / T = 1.200 \times 10^{+00}$$

$$L/R = 3.000 \times 10^{+00}$$

$$\bar{T}_Y / T = 3.000 \times 10^{-01}$$

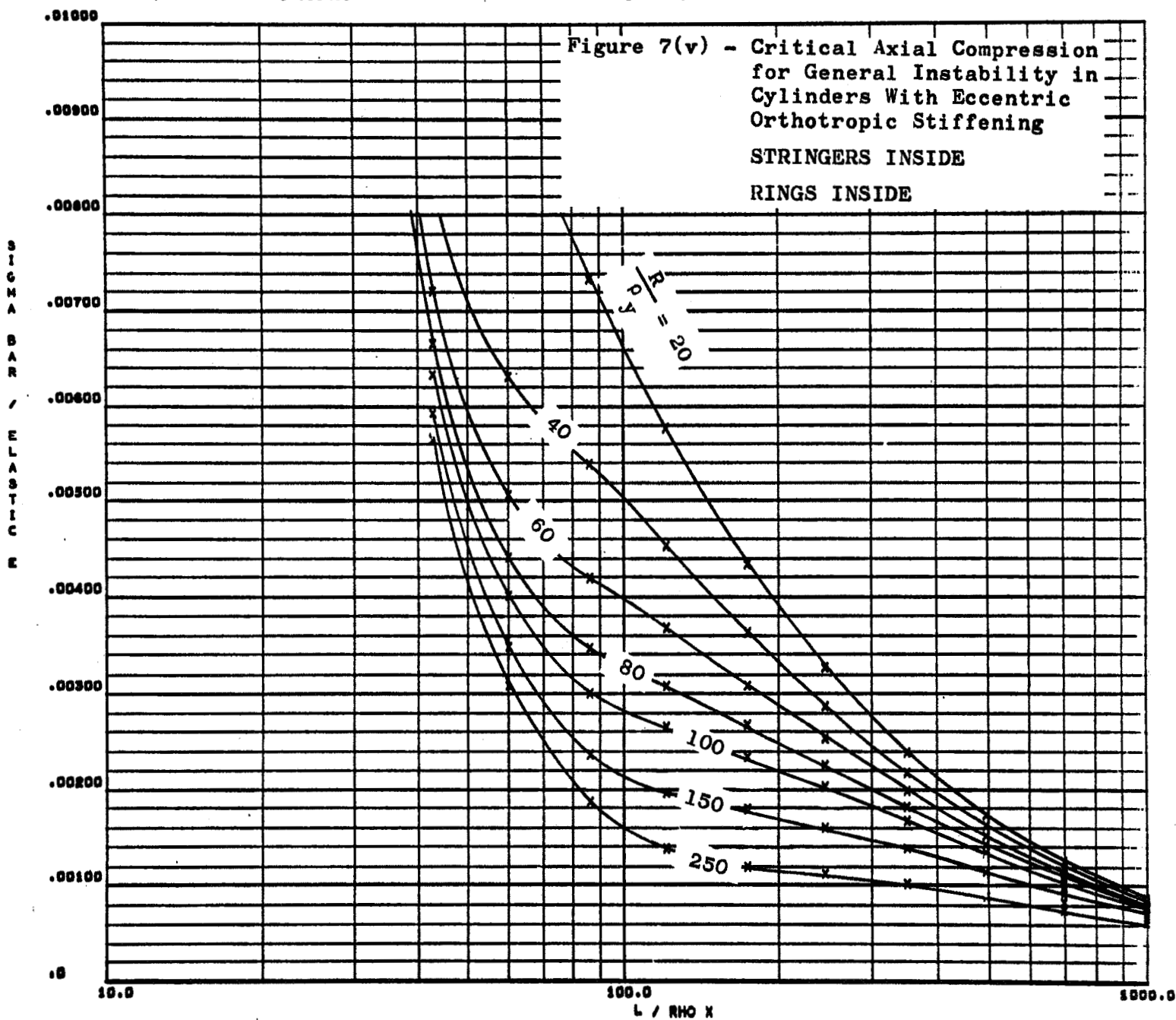




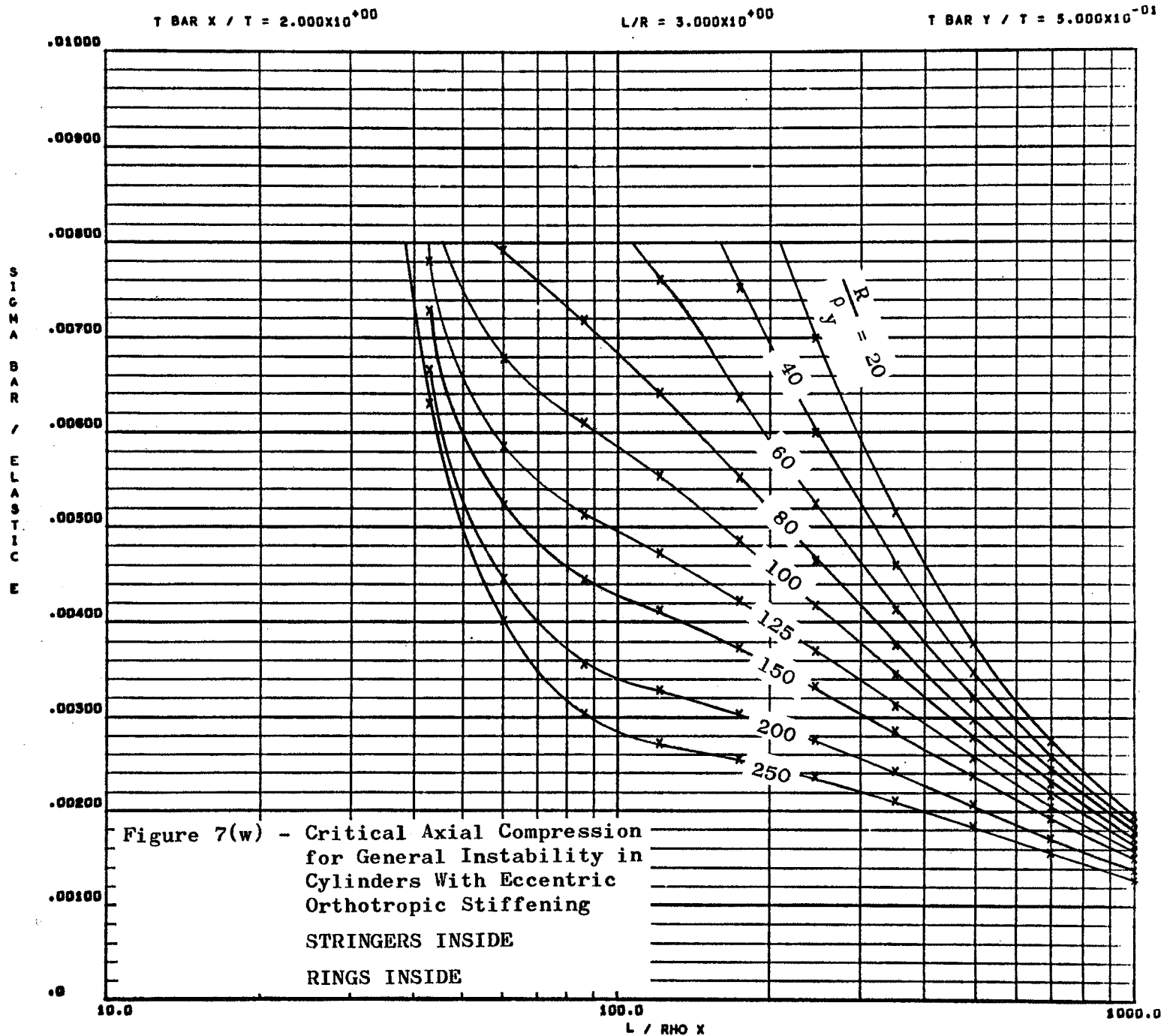
$$T \text{ BAR } X / T = 2.000 \times 10^{-00}$$

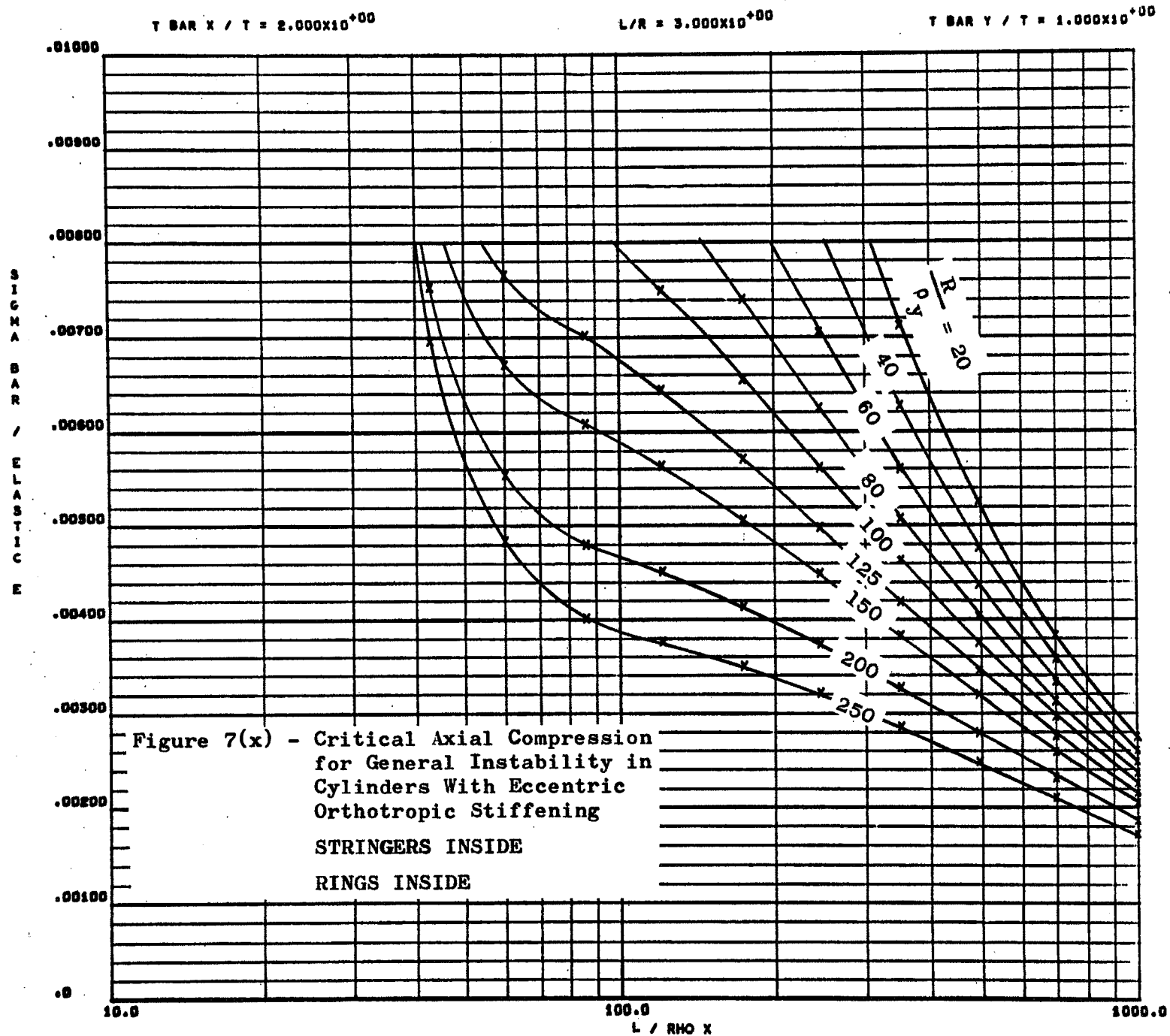
$$L/R = 3.000 \times 10^{-00}$$

$$T \text{ BAR } Y / T = 1.000 \times 10^{-01}$$



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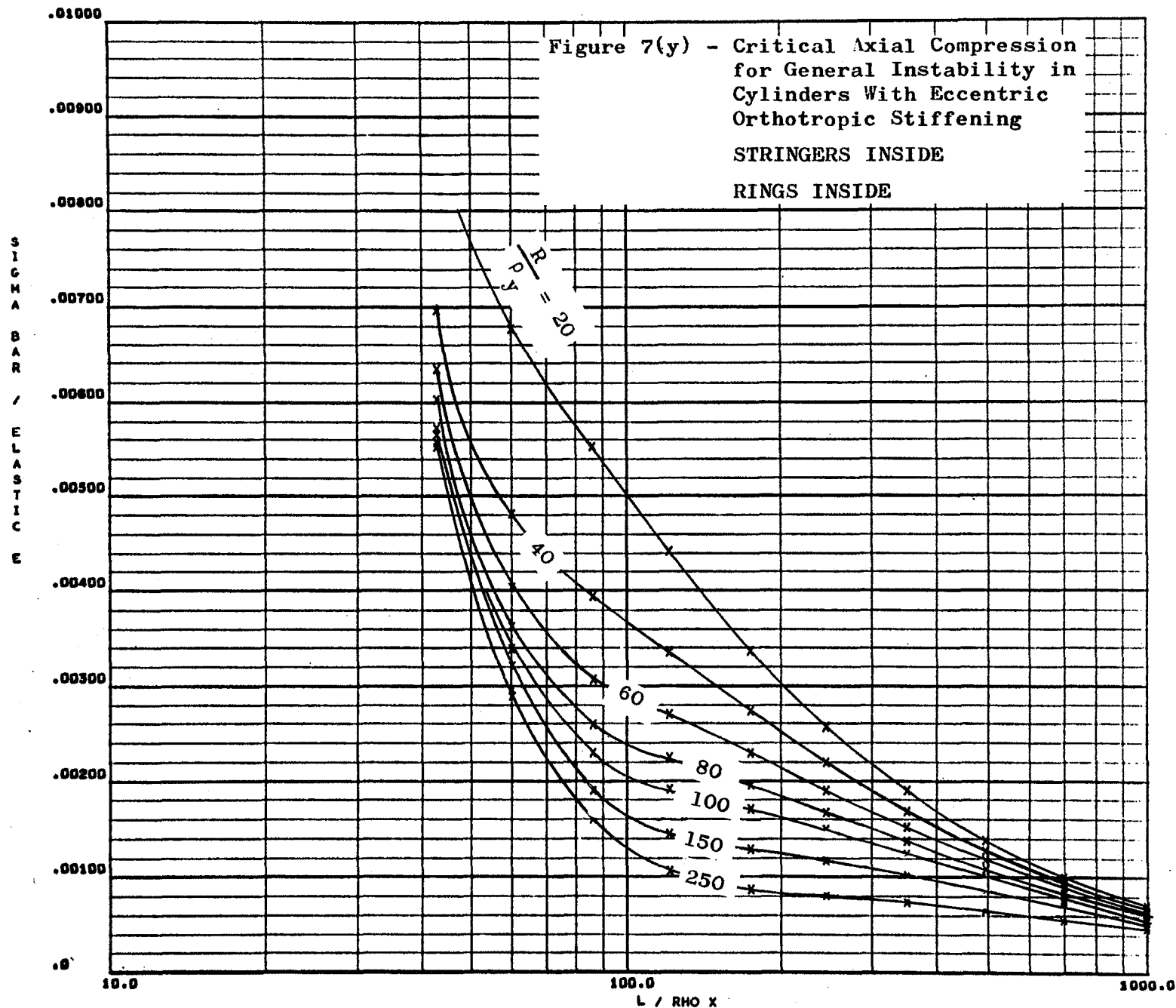




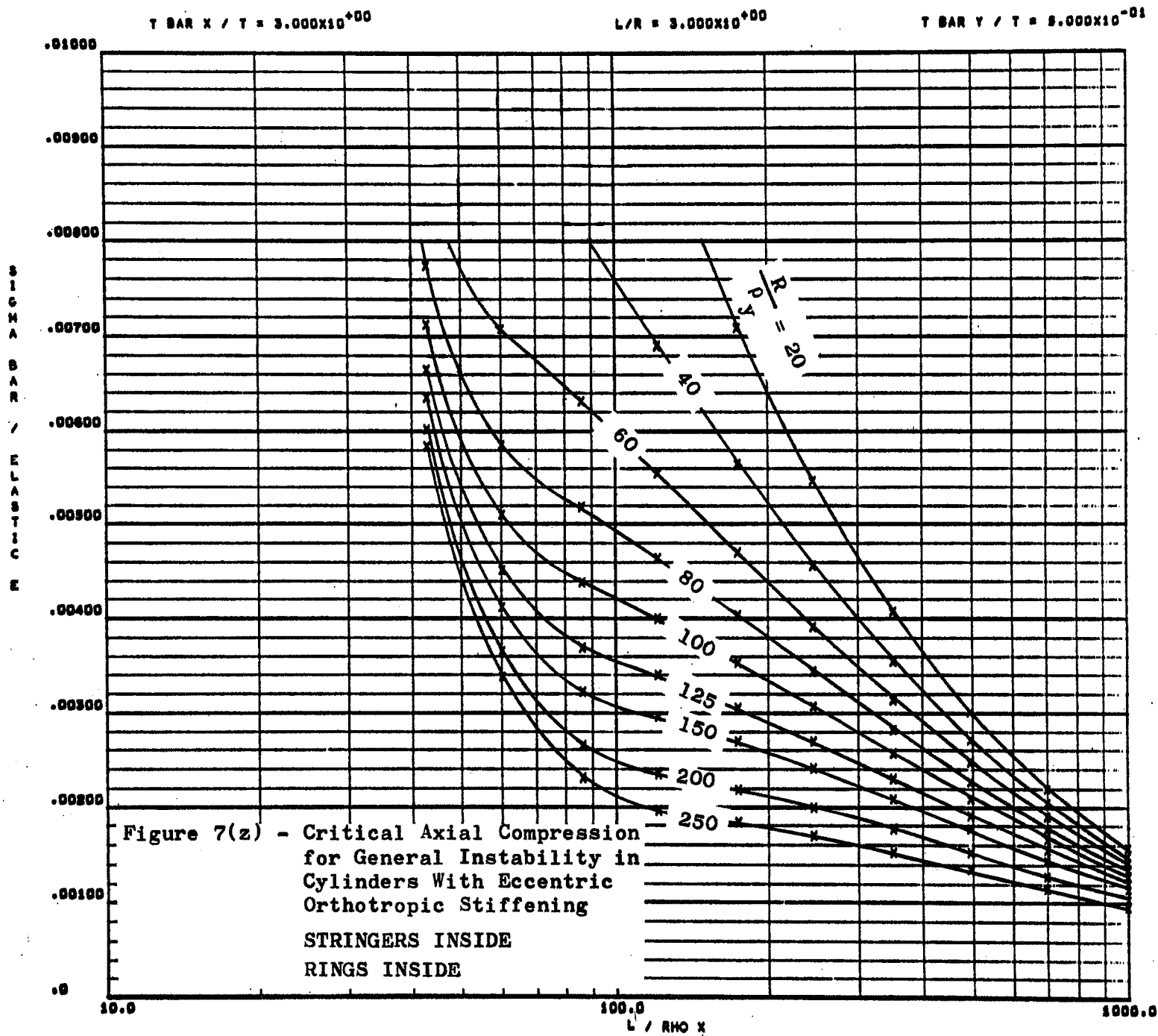
$$T \text{ BAR } X / T = 3.000 \times 10^{+00}$$

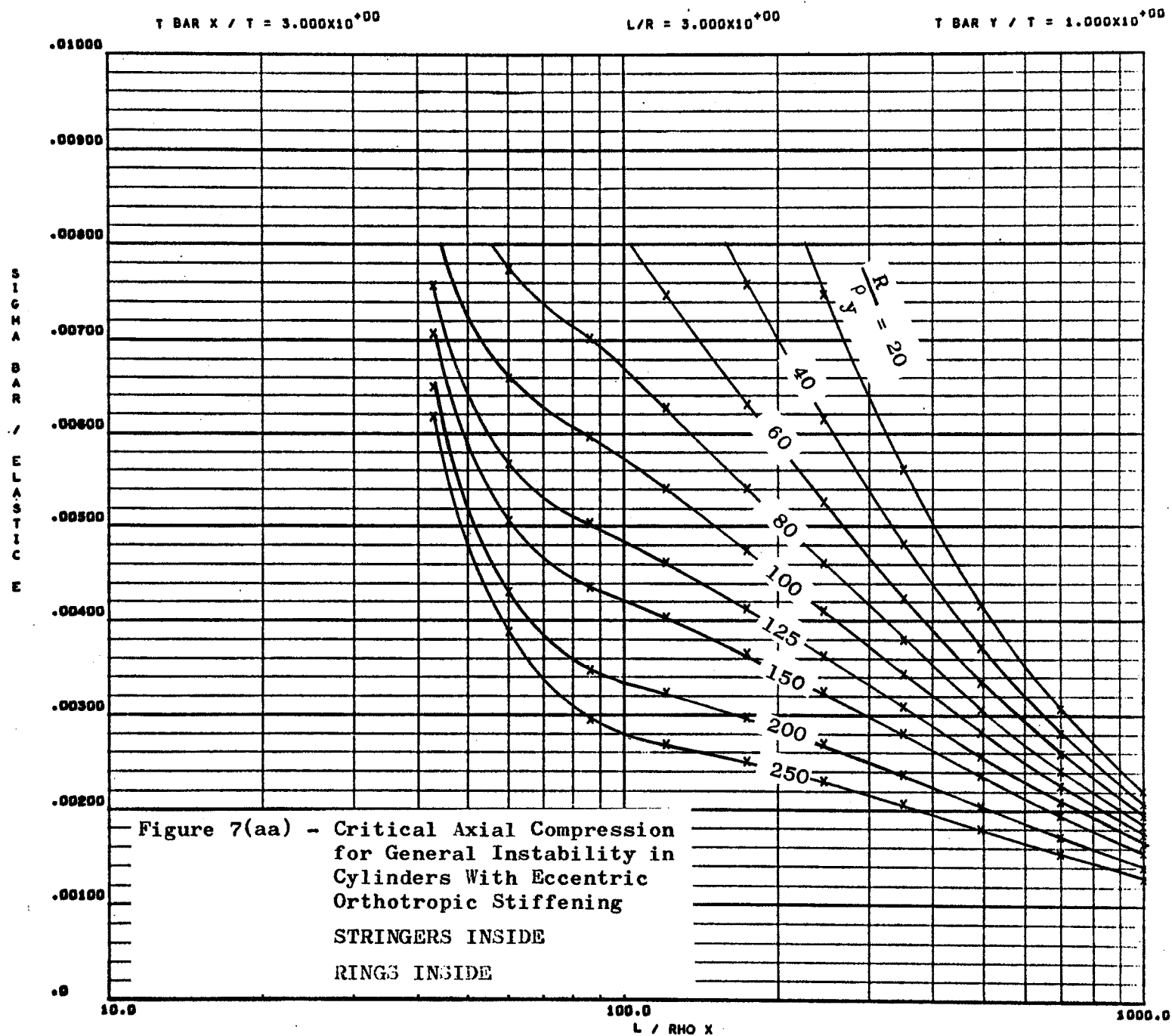
$$L/R = 3.000 \times 10^{+00}$$

$$T \text{ BAR } Y / T = 1.000 \times 10^{-01}$$



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SECTION 5

ELASTIC CONSTANTS

The digital computer program of SECTION 6 includes an option which allows for the input of elastic constants and eccentricity coupling constants. This feature was incorporated to provide the engineer with a more flexible analysis method than is given by the curves of SECTION 4. However, in order to make use of this capability, one must first compute appropriate values for the A_{ij} 's, D_{ij} 's, and C_{ij} 's. Some recommended formulas for these constants are listed in Table III. The tabulated formulas are simplified expressions suitable for application to most practical stiffened configurations (not including waffle-type structures). To be rigorous, more complicated expressions would be required. All of the given formulas apply only where the behavior is elastic. For cases where the buckling stress exceeds the proportional limit of the stress-strain curve, the stringer (or corrugation) and skin stiffnesses should be modified to reflect tangent modulus influences.

In addition to the elastic constants and eccentricity coupling constants, Table III also lists recommended formulas for the computation of the quantities \bar{t}_x , \bar{t}_y , ρ_x , and ρ_y which must be computed if one is to make use of the curves given in SECTION 4 or the RATIO option of digital computer program 4267 (see SECTION 6).

To fully understand Table III, it is helpful to note that the A_{ij} 's and D_{ij} 's arise out of mathematical integrations involving the distribution of the composite wall material about the appropriate centroidal surface. Note that the centroidal surface has a curvature of its own. Therefore, the related material distribution is equivalent to that which exists about the centroidal plane of the flat plate obtained by unfolding the composite circular shell wall into a flat configuration. All influences of curvature in this regard are inherent in the basic shell equations into which the A_{ij} 's and D_{ij} 's are substituted.

Table III applies only to cases where no buckling of the isotropic skin panels and no local buckling of the stringers occur prior to overall instability. In addition, it is assumed that the stringers are spaced sufficiently close together to justify the assumption that all of the skin material is fully effective. In cases where these several conditions do not prevail, it becomes necessary to introduce effective-width concepts to modify the information given in the table and the notes which follow it.

TABLE III - Recommended Formulas for Fundamental Constants of Circular Cylinders Having Both Stringers and Rings

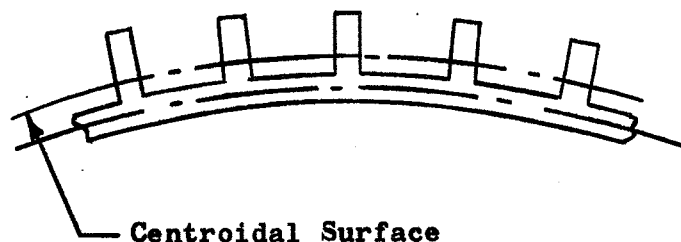
CASE	CONFIGURATION	\bar{I}_x	\bar{I}_y	P_x	P_y	A_{11}	A_{22}	A_{12}	A_{33}	D_{11}	D_{22}	D_{12}	D_{33}	C_{11}	C_{22}	C_{12}	C_{21}
A		$\left(\frac{b^3 h}{6} + t \right)$															
			$\frac{A r^3}{A}$	$\sqrt{\frac{I_x}{r^3}}$	$\sqrt{\frac{I_y}{r^3}}$	$\frac{1}{EI_x}$	$\frac{1}{EI_y}$	0	$\frac{1}{Gt}$	EI_x	$\frac{EI r^3}{a}$	0	$\frac{Gt^3}{12}$	\bar{I}_x (Positive for internal stringers; Negative for external stringers)	\bar{I}_y (Positive for internal rings; Negative for external rings)	0	0
		$\left(\frac{A}{b} + t \right)$															
	ETC.																
B		$\left[\frac{(2d_1)}{2\pi R} t_c + t \right]$	$\frac{A r^3}{a}$	$\sqrt{\frac{I_x}{r^3}}$	$\sqrt{\frac{I_y}{r^3}}$	$\frac{1}{EI_x}$	$\frac{1}{EI_y}$	0	$\frac{1}{G \left[\frac{2\pi R}{(2d_1)} t_c + t \right]}$	EI_x	$\frac{EI r^3}{a}$	0	$\frac{G}{12} \left[\frac{2\pi R}{(2d_1)} t_c^3 + t^3 \right]$	\bar{I}_x (Positive for internal corrugation; Negative for external corrugation)	\bar{I}_y (Positive for internal rings; Negative for external rings)	0	0
C		$\left[\frac{(2d_1)}{2\pi R} t_c \right]$	$\frac{A r^3}{a}$	$\sqrt{\frac{I_x}{r^3}}$	$\sqrt{\frac{I_y}{r^3}}$	$\frac{1}{EI_x}$	$\frac{1}{EI_y}$	0	$\frac{1}{G \left[\frac{2\pi R}{(2d_1)} t_c \right]}$	EI_x	$\frac{EI r^3}{a}$	0	$\frac{G}{12} \left[\frac{2\pi R}{(2d_1)} t_c^3 \right]$	0	\bar{I}_y (Positive for internal rings; Negative for external rings)	0	0

Notes for TABLE III

- (a) For convenience, all of the non-corrugated configurations shown here only depict the case of external stringers. In addition, only internal rings are shown. However, all of the formulas for \bar{t}_x , \bar{t}_y , ρ_x , ρ_y , the A_{ij} 's, and the D_{ij} 's apply equally well for other orientations of the stiffeners (inside vs. outside). On the other hand, the signs of the C_{11} and C_{22} values depend on the stiffener locations (\bar{z}_x is positive for internal stringers and negative for external stringers; \bar{z}_y is positive for internal rings and negative for external rings).
- (b) The quantity \bar{t}_x is the wall thickness for a monocoque circular cylinder of the same radius as the middle surface of the stiffened-cylinder basic skin, and of the same total cross-sectional area as the actual composite stiffened wall including both skin and stringers. The cross section referred to here is obtained by passing a plane through the cylinder, normal to the axis of revolution.
- (c) The quantity A_s is the cross-sectional area of a single stringer and does not include any of the basic cylindrical skin. The cross section referred to here is obtained by passing a plane through the cylinder, normal to the axis of revolution.
- (d) The symbol (Σd_i) is used to denote the total peripheral length of the corrugation center-line for the wave-type cross section obtained by passing a plane through the cylinder, normal to the axis of revolution. Hence, $(\Sigma d_i) > 2\pi R$ and the total area for the stated corrugation cross section may be taken equal to $(\Sigma d_i)(t_c)$.

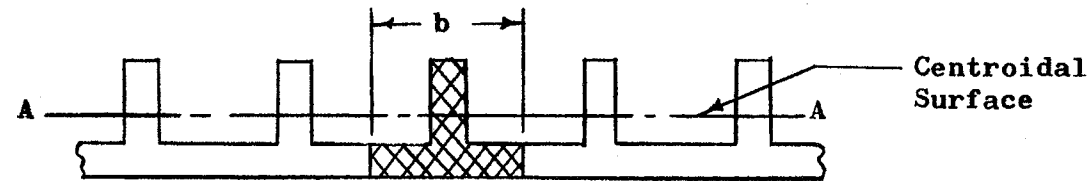
Notes for TABLE III (Continued)

- (e) The quantity A_{r+s} is the cross-sectional area of a single ring augmented by an effective width of skin which is considered to essentially behave as part of the ring. The user must apply engineering judgement in selecting the effective width to be used here. The cross section referred to lies in a radial plane.
- (f) The quantity A_r is the cross-sectional area of a single ring and does not include any effective width of skin. The cross section referred to here lies in a radial plane.
- (g) The quantity a is the spacing between rings (assumed to be uniform throughout the structure).
- (h) The quantity \bar{I}_x is the local longitudinal centroidal running moment of inertia for the flat configuration obtained by unfolding the entire composite circular shell wall. For example, consider the case of a cylinder having a local wall cross section of the type



Notes for TABLE III (Continued)

In such a case, one must consider the unfolded geometry shown below.



After computing I_{A-A} for the cross-hatched area shown, \bar{I}_x is found as follows:

$$\bar{I}_x = \frac{I_{A-A}}{b}$$

- (i) The quantity \bar{I}_{r+s} is the centroidal moment of inertia for the cross section of a single ring augmented by an effective width of skin which is considered to essentially behave as part of the ring. The user must apply engineering judgement in selecting the effective width to be used here. The cross section referred to lies in a radial plane.

Notes for TABLE III (Continued)

- (j) The quantity \bar{I}_r is the centroidal moment of inertia for the cross section of a single ring and does not include any effective width of skin. The cross section referred to here lies in a radial plane.
- (k) The term $(1-\nu^2)$ has been omitted from the formulas for D_{11} and D_{22} since the specified configurations provide incomplete restraint to the related anticlastic bending (see GLOSSARY, Volume I [1]).
- (l) The several values which have been set equal to zero all involve the available mechanisms for interplay that are dependent upon Poisson's-ratio influences. For usual stiffened constructions, such interplay is of a minimal nature since it occurs primarily in the localized regions where the stiffeners intersect. However, when the stringer and ring spacings are very small (as for waffle configurations) and/or the contributions from stiffener rigidities approach that of the skin (also as for waffle configurations), non-zero values should be used for A_{12} , D_{12} , C_{12} , and C_{21} . However, no recommendations are made here as to practical formulations to be used for the computation of such values.

SECTION 6

DIGITAL COMPUTER PROGRAM

This section presents the essential features of General Dynamics Convair digital computer program numbered 4267. This program was developed for the analysis of general instability in axially compressed circular cylinders having eccentric stringers and rings. The solution is based upon the theoretical considerations presented in SECTION 2. The output can be obtained in the form of automatically plotted data or single-point solutions, as desired. All of the buckling curves presented in SECTION 4 were obtained by using the automatic plotting option of the program. The input format is shown in Figure 8. Symbols are listed in Table IV. A detailed, card-by-card description of the input follows below. Runs may be stacked. For further information concerning this program one may refer to the detailed derivations and basic logic presented in reference 15.

CARD TYPE 1: One card per run.

Enter PROBLEM IDENTIFICATION anywhere in columns 1-60.

Alphanumeric characters.

CARD TYPE 2: One card per run.

Enter INPUT OPTION (RATIO or STIFF) in columns 1-5.

This option permits the user to choose between alternative formats for cards other than types 1, 2, and 3.

Enter NO OF CASES as right adjusted integer in columns 6-10 (I5).

Enter POISSON'S RATIO (ν) in columns 11-15 (F5).

Enter E/G (Young's modulus \div Modulus of Elasticity in Shear) in columns 16-20 (F5).

PROBLEM			GENERAL INSTABILITY OF ECCENTRICALLY STIFFENED CYLINDERS																																			RWO		EWO																															
CODER			F.A. MUSCHA																																			PAGE		OF																															
EXT.			DATE																																			PAGE		OF																															
1	1	PROBLEM IDENTIFICATION																																																																					
2	2																																																																						
3	3	INPUT NO OF POISSON'S (E/G)										LOWER					UPPER					P ₀					C ₁₂					C ₂₁					A ₁₂					N _R					DUMP																								
4	4	OPTION CASES										RATIO					C _u					C _u					C ₀					C ₀					NRRHOY					OPTION					OPTION					OPTION					OPTION					N _R					DUMP				
5	5																																																																						
6	6																																																																						
7	7	MAX ORDINATE										SCREENING										MIN NO CIRCUMF																																																	
8	8	FOR PLOTS										CUT-OFF										HALF-WAVES																																																	
9	9																																																																						
10	10																																																																						
11	11	(RRHOY) ₁										(RRHOY) ₂										(RRHOY) ₃														(RRHOY) _{NRRHOY-1}					(RRHOY) _{NRRHOY}																			
12	12																																																																						
13	13	CASE NO										OUTPUT					(E _u /E)					(L/R)					(E _y /E)					C _{0x}					C _{0y}					C _{0y}					C _{0y}																								
14	14																																																																						
15	15	STRENGER										RING										BETA																																																	
16	16	LOCATION										LOCATION										FACTOR																																																	
17	17																																																																						
18	18																																																																						
19	19	(L/P _u)										(R/P _y)										(L/P _{2y})										(C ₁₁ /R)					(C ₂₂ /R)					(C ₁₂ /R)					(C ₂₁ /R)																								
20	20																																																																						
21	21																																																																						
22	22																																																																						
23	23																																																																						
24	24																																																																						
25	25	CASE NO										A ₁₁					A ₂₂					A ₁₂					A ₂₀					C ₁₁					C ₂₂					R																													
26	26											D ₁₁					D ₂₂					D ₂₂					D ₂₀					C ₁₂					C ₂₁					L																													
27	27																																																																						
28	28																																																																						
29	29																																																																						
30	30																																																																						

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Figure 8 - Input Format - Program 4267

Enter LOWER C_α in columns 21-25 (F5). Output is listed for critical $m (=m^*)$ and also for $m=(\text{LOWER } C_\alpha) \times m^*$. The value $(\text{LOWER } C_\alpha) = 0.99$ will usually be suitable.

Enter UPPER C_α in columns 26-30 (F5). Output is listed for critical $m (=m^*)$ and also for $m = (\text{UPPER } C_\alpha) \times m^*$. The value $(\text{UPPER } C_\alpha) = 1.01$ will usually be suitable.

Enter LOWER C_β in columns 31-35 (F5). Output is listed for critical $\beta (= \beta^*)$ and also for $\beta = (\text{LOWER } C_\beta) \times \beta^*$. The value $(\text{LOWER } C_\beta) = 0.99$ will usually be suitable.

Enter UPPER C_β in columns 36-40 (F5). Output is listed for critical $\beta (= \beta^*)$ and also for $\beta = (\text{UPPER } C_\beta) \times \beta^*$. The value $(\text{UPPER } C_\beta) = 1.01$ will usually be suitable.

Enter NRRHOY (number of R/ρ_y ratios to be included in plots and/or tables which result from automatic sequencing operations) as right adjusted integer in columns 41-45 (I5). Will be left blank when only point solutions are to be obtained.

Enter η_p OPTION as right adjusted integer in columns 46-50 (I5). Always insert the number 1 here except where

- (a) it is desired to eliminate the ρ_{xy} contribution in RATIO-option point solutions
- or
- (b) it is desired to eliminate the η_p contribution in STIFF-option point solutions.

In the latter two instances leave columns 46-50 blank.

Enter C_{12} OPTION as right adjusted integer in columns 51-55 (I5). Always insert the number 1 here except where it is desired to eliminate the C_{12} contribution in point solutions. (For other types of solutions the program always assumes $C_{12} = 0$). For those point solutions where one chooses to ignore the C_{12} influence, columns 51-55 are to be left blank.

Enter C_{21} OPTION as right adjusted integer in columns 56-60 (I5). Always insert the number 1 here except where it is desired to eliminate the C_{21} contribution in point solutions. (For other types of solutions, the program always assumes $C_{21} = 0$). For those point solutions where one chooses to ignore the C_{21} influence, columns 56-60 are to be left blank.

Enter A_{12} OPTION as right adjusted integer in columns 61-65 (I5). Always insert the number 1 here except where it is desired to eliminate the A_{12} contribution in any type of solution. In the latter case, columns 61-65 are to be left blank.

Enter N_R as a right adjusted integer in columns 66-70 (I5). This input constitutes the number of refinement cycles used to improve the accuracy of final computed values. Whenever (BETA FACTOR) ≤ 1.05 , the value $N_R=5$ should usually be satisfactory. Each single refinement cycle essentially cuts the final β screening increment in half.

Enter DUMP OPTION as a right adjusted integer in columns 71-75 (I5). Insert the number 1 whenever supplementary diagnostic output data is to be printed out. Otherwise, leave blank.

CARD TYPE 3: One card per run.

Enter MAX ORDINATE FOR PLOTS in columns 1-10 (E10.5). Any of the following values may be inserted here:

.0005	.001	.006
.0006	.002	.007
.0007	.003	.008
.0008	.004	.009
.0009	.005	.010

This entry is left blank when no plots are to be made.

Enter SCREENING CUT-OFF in columns 11-20 (E10.5).

This is a cut-off value used in the minimization process and must be set greater than the output critical strain. Whenever the computed critical strain exceeds the SCREENING CUT-OFF value, the printed results cannot be believed. In such cases, one should increase the input SCREENING CUT-OFF value and rerun the program. In general one will have little interest in critical strains above .010. Therefore, this value will usually be a suitable choice for the SCREENING CUT-OFF.

Enter MIN NO CIRCUMF HALF-WAVES in columns 21-30 (E10.5).

This is the minimum number of circumferential half-waves considered to be permissible for non-axisymmetric buckle patterns. A value of 2.0 should usually be inserted here.

CARD TYPE 4 (RATIO OPTION):

There will be $\text{NRRHOY}/8$ (rounded to the higher whole number) cards per run. However, when $\text{NRRHOY} = 0$, no cards of this type are required.

Enter RRHOY (R/ρ_y) values, with a maximum of 8 to a card (8 E10.5).

CARD TYPE 5 (RATIO OPTION):

There will be one of these cards for each case.

Enter CASE NO as right adjusted integer in columns 1-5 (I5).

Enter OUTPUT OPTION as right adjusted integer in columns 6-10 (I5).

- 1 = Tables with no plots
- 2 = Tables plus plots
- 3 = Plots with no tables
- 4 = Point solution

Enter the thickness ratio (\bar{t}_x/t) in columns 11-20 (E10.5).

Enter the (Length/Radius) ratio L/R in columns 21-30 (E10.5).

Enter the thickness ratio (\bar{t}_y/t) in columns 31-40 (E10.5).

Enter the geometric factor C_{px} in columns 41-50 (E10.5).

This factor is defined by equation (2-26) and is not used for point solutions. Hence, when OUTPUT OPTION = 4, leave this entry blank. The value of $C_{px} = .33$ will usually be reasonable for practical skin-stringer combinations.

Enter the geometric factor C_{hx} in columns 51-60 (E10.5). This factor is defined by equation (2-25) and is not used for point solutions. Hence, when OUTPUT OPTION = 4, leave this entry blank. When it is desired that the computer employ the equation

$$C_{hx} = \left(\frac{1}{2}\right) \frac{\left[\left(\frac{\bar{t}_x}{t}\right) - 1\right]}{\left(\bar{t}_x/t\right)} \quad (6-1)$$

insert the left adjusted word AUTOMATIC in columns 51-60.

Enter the geometric factor C_{py} in columns 61-70 (E10.5). This factor is defined by equation (2-30) and is not used for point solutions. Hence, when OUTPUT OPTION = 4, leave this entry blank. The value $C_{py} = .40$ will usually be reasonable for practical skin-ring combinations.

Enter the geometric factor C_{hy} in columns 71-80 (E10.5). This factor is defined by equation (2-29) and is not used for point solutions. Hence, when OUTPUT OPTION = 4, leave this entry blank. The value $C_{hy} = .425$ will usually be reasonable for practical skin-ring combinations.

CARD TYPE 6 (RATIO OPTION):

There will be one of these cards for each case.

Enter STRINGER LOCATION (INSIDE, OUTSIDE, SYMMETRIC) as left adjusted word in columns 1-10.

Enter RING LOCATION (INSIDE, OUTSIDE, SYMMETRIC) as left adjusted word in columns 11-20.

Enter BETA FACTOR in columns 21-30 (E10.5). This is a stepping factor used in the minimization process. That is, screening is performed involving β values computed from

$$\beta_{i+1} = \frac{\beta_i}{(\text{BETA FACTOR})} \quad (6-2)$$

The value (BETA FACTOR) = 1.02 should be suitable for most applications.

CARD TYPE 7 (RATIO OPTION):

This card only required for point solutions (OUTPUT OPTION = 4).

Enter the longitudinal slenderness ratio L/ρ_x in columns 1-10 (E10.5).

Enter the hoop slenderness ratio R/ρ_y in columns 11-20 (E10.5).

Enter the artificial slenderness ratio L/ρ_{xy} in columns 21-30 (E10.5).

Enter the eccentricity-dependent ratio C_{11}/R in columns 31-40 (E10.5).

Enter the eccentricity-dependent ratio C_{22}/R in columns 41-50 (E10.5).

Enter the eccentricity-dependent ratio C_{12}/R in columns 51-60 (E10.5).

Enter the eccentricity-dependent ratio C_{21}/R in columns 61-70 (E10.5).

CARD TYPE 4 (STIFF OPTION):

There will be one of these cards for each case.

Enter CASE NO as right adjusted integer in columns 1-10 (I10).

Enter the elastic constant A_{11} in columns 11-20 (E10.5).

Enter the elastic constant A_{22} in columns 21-30 (E10.5).

Enter the elastic constant A_{12} in columns 31-40 (E10.5).

Enter the elastic constant A_{33} in columns 41-50 (E10.5).

Enter the eccentricity coupling constant C_{11} in columns 51-60 (E10.5).

Enter the eccentricity coupling constant C_{22} in columns 61-70 (E10.5).

Enter R (the radius to the middle surface of the basic cylindrical skin) in columns 71-80 (E10.5).

CARD TYPE 5 (STIFF OPTION):

There will be one of these cards for each case.

Enter the elastic constant D_{11} in columns 11-20 (E10.5).

Enter the elastic constant D_{22} in columns 21-30 (E10.5).

Enter the elastic constant D_{12} in columns 31-40 (E10.5).

Enter the elastic constant D_{33} in columns 41-50 (E10.5).

Enter the eccentricity coupling constant C_{12} in columns 51-60 (E10.5).

Enter the eccentricity coupling constant C_{21} in columns 61-70 (E10.5).

Enter the overall length L (this is NOT the ring spacing) in columns 71-80 (E10.5).

CARD TYPE 6 (STIFF OPTION):

There will be one of these cards for each case.

Enter BETA FACTOR in columns 11-20 (E10.5). This is a stepping factor used in the minimization process. That is, screening is performed involving β values computed from

$$\beta_{i+1} = \frac{\beta_i}{(\text{BETA FACTOR})} \quad (6-3)$$

The value (BETA FACTOR) = 1.02 should be suitable for most applications.

A sample input coding form is shown in Figure 9.

The program output consists of a listing and/or plots depending upon the options selected. A sample output listing for

INPUT OPTION = RATIO

OUTPUT OPTION = 1

is shown in Figure 10. Typical plots are given in SECTION 4. A basic flow diagram for the program is presented as Figure 11 and a Fortran listing of the program is shown in Table V.

TABLE IV - Program 4267 Notation

<u>Program Notation</u>	<u>Report Notation</u>	<u>Description</u>
TITLE	----	Problem identification.
RAORST	Input Option	RATIO or STIFF.
NCASES	----	Number of cases.
POISR	ν	Poisson's ratio.
EDSIG	(E/G)	Ratio of Young's modulus to modulus of elasticity in shear.
CALPHL	Lower C_α	See description of input for CARD TYPE 2.
CALPHU	Upper C_α	See description of input for CARD TYPE 2.
CBETAL	Lower C_β	See description of input for CARD TYPE 2.
CBETAU	Upper C_β	See description of input for CARD TYPE 2.
NRRHOY	NRRHOY	The number of R/ρ ratios to be included in y plots and/or tables which result from automatic sequencing operations.
IETAP	η_p Option	See description of input for CARD TYPE 2.
IC12	C_{12} Option	See description of input for CARD TYPE 2.
IC21	C_{21} Option	See description of input for CARD TYPE 2.
IA12	A_{12} Option	See description of input for CARD TYPE 2.
NR	N_R	See description of input for CARD TYPE 2.
IDUMP	Dump Option	See description of input for CARD TYPE 2.

TABLE IV - Program 4267 Notation
(Continued)

<u>Program Notation</u>	<u>Report Notation</u>	<u>Description</u>
ORDMAX	Max Ordinate For Plots	Uppermost grid line for plots.
SCREEN	Screening Cut-Off	See description of input for CARD TYPE 3.
CHWMIN	Min no. circumf. half-waves	Cut-off value used in mini- mization process. See description of input for CARD TYPE 3.
RDRHOY	----	R/p ratios to be in plots or tables.
NCASE	----	Case number.
ITYPEO	Output Option	1 = Tables only 2 = Tables plus plots 3 = Plots only 4 = Point solution
THICKX	\bar{t}_x/t	Thickness ratio.
FLGRAD	L/R	Length-to-radius ratio.
THICKY	\bar{t}_y/t	Thickness ratio.
CRHOX	$C_{\rho x}$	Geometric factor defined by equation (2-26).
CHX	C_{hx}	Geometric factor defined by equation (2-25).
CRHOY	$C_{\rho y}$	Geometric factor defined by equation (2-30).
CHY	C_{hy}	Geometric factor defined by equation (2-29).
STRING	Stringer Location	Stringer location (INSIDE, OUTSIDE, SYMMETRIC).

TABLE IV - Program 4267 Notation
(Continued)

<u>Program Notation</u>	<u>Report Notation</u>	<u>Description</u>
RINGL	Ring Location	Ring location (INSIDE, OUTSIDE, SYMMETRIC).
BETAF	Beta Factor	See description of input for CARDS TYPE 6.
FLRHOX	L/ρ_x	Longitudinal slenderness ratio.
FLRHOY	R/ρ_y	Circumferential slenderness ratio for point solution only.
FLROXY	L/ρ_{xy}	Artificial slenderness ratio.
C11R	C_{11}/R	Eccentricity coupling ratio.
C22R	C_{22}/R	Eccentricity coupling ratio.
C12R	C_{12}/R	Eccentricity coupling ratio.
C21R	C_{21}/R	Eccentricity coupling ratio.
A11	A_{11}	Elastic constant.
A22	A_{22}	Elastic constant.
A12	A_{12}	Elastic constant.
A33	A_{33}	Elastic constant.
C11	C_{11}	Eccentricity coupling constant.
C22	C_{22}	Eccentricity coupling constant.
R	R	Radius.
D11	D_{11}	Elastic constant.
D22	D_{22}	Elastic constant.
D12	D_{12}	Elastic constant.

TABLE IV - Program 4267 Notation
(Continued)

<u>Program Notation</u>	<u>Report Notation</u>	<u>Description</u>
D33	D_{33}	Elastic constant.
C12	C_{12}	Eccentricity coupling constant.
C21	C_{21}	Eccentricity coupling constant.
FLENGT	L	Overall length of cylinder.

FORTRAN CODING AND DATA FORM

PROBLEM		CODER		EXT.		DATE		PAGE		OF		
GENERAL INSTABILITY OF ECCENTRICALLY STEFFENED CYLINDERS								1		1		
1	1	SAMPLE PROBLEM										1
2	2	RATIO	1	.3	2.6	.99	1.01	.99	1.01	1	1	5
3	3		.01		2.0							
4	4	200.0										
5	5	1	1	3.0	2.0	.5	.33	AUTOMATIC	.4		.425	
6	6	OUTSIDE	INSIDE		1.02							

CARD TYPE:

GENERAL DYNAMICS CONVAIR DIVISION

6-15

GDC-DDG-67-006

GENERAL INSTABILITY OF ECCENTRICALLY STIFFENED CYLINDERS 4267

SAMPLE PROBLEM

INPUT OPTION RATIO	NUMBER OF CASES 1	POISSONS RATIO 0.300	E/G 2.600	LOWER C SUB ALPHA 0.990	UPPER C SUB ALPHA 1.010	LOWER C SUB BETA 0.990	UPPER C SUB BETA 1.010	NRHOY 1	ETA SUB P OPTION 1	C12 OPTION 1	C21 OPTION 1	A12 OPTION 1
--------------------------	-------------------------	----------------------------	--------------	-------------------------------	-------------------------------	------------------------------	------------------------------	------------	--------------------------	--------------------	--------------------	--------------------

MAX ORDINATE
FOR PLOTS

-0.

SCREENING
CUT-OFF

1.00000E-02

MIN NO CIRC
HALF-WAVES

2.00

N SUB R

5

R/RHO Y RATIOS FOR AUTOMATIC SEQUENCING

2.00000E 02

CASE NUMBER	OUTPUT OPTION	T BAR X/T	L/R	T BAR Y/T	C SUB RHO X	C SUB HX	C SUB RHO Y	C SUB HY
1	1	3.00000E 00	2.00000E 00	5.00000E-01	3.30000E-01	3.33333E-01	4.00000E-01	4.25000E-01

STRINGER LOCATION

OUTSIDE

RING LOCATION

INSIDE

BETA FACTOR

1.02000E 00

R/RHO Y

2.00000E 02

L/RHO X	CRITICAL LOWER CASE M	CRITICAL LOWER CASE N	CRITICAL SIGMA BAR/E	LL SIGMA BAR/E	LH SIGMA BAR/E	HL SIGMA BAR/E	HH SIGMA BAR/E	AXISYM SIGMA BAR/E
3.00000E 01	1.00000E 00	5.68383E 00	2.59964E-02	2.56541E-02	2.56608E-02	2.63472E-02	2.63407E-02	5.44331E-02
4.25000E 01	1.00000E 00	5.72265E 00	1.54430E-02	1.52721E-02	1.52776E-02	1.56212E-02	1.56155E-02	3.84234E-02
6.00000E 01	1.00000E 00	5.66978E 00	9.90249E-03	9.81826E-03	9.82342E-03	9.99283E-03	9.98782E-03	2.72166E-02
8.60000E 01	1.00000E 00	5.59234E 00	6.79592E-03	6.75770E-03	6.76227E-03	6.84004E-03	6.83505E-03	1.89883E-02
1.21500E 02	1.00000E 00	5.51530E 00	5.15497E-03	5.13917E-03	5.14397E-03	5.17599E-03	5.17157E-03	1.34403E-02
1.75000E 02	1.00000E 00	5.45213E 00	4.17198E-03	4.16858E-03	4.17285E-03	4.18089E-03	4.17611E-03	9.33139E-03
2.45000E 02	1.53299E 00	6.37875E 00	3.57925E-03	3.57950E-03	3.58316E-03	3.58323E-03	3.57948E-03	6.66528E-03
3.50000E 02	2.77085E 00	7.63813E 00	2.80974E-03	2.80990E-03	2.81228E-03	2.81231E-03	2.80990E-03	4.66569E-03
4.95000E 02	3.95780E 00	8.29345E 00	2.10377E-03	2.10393E-03	2.10554E-03	2.10557E-03	2.10392E-03	3.29898E-03
7.00000E 02	5.21559E 00	8.73586E 00	1.53472E-03	1.53486E-03	1.53600E-03	1.53601E-03	1.53487E-03	2.33285E-03
1.00000E 03	6.64537E 00	9.07263E 00	1.09353E-03	1.09366E-03	1.09446E-03	1.09448E-03	1.09365E-03	1.63299E-03

Figure 10 - Sample Output Listing - Program 4267

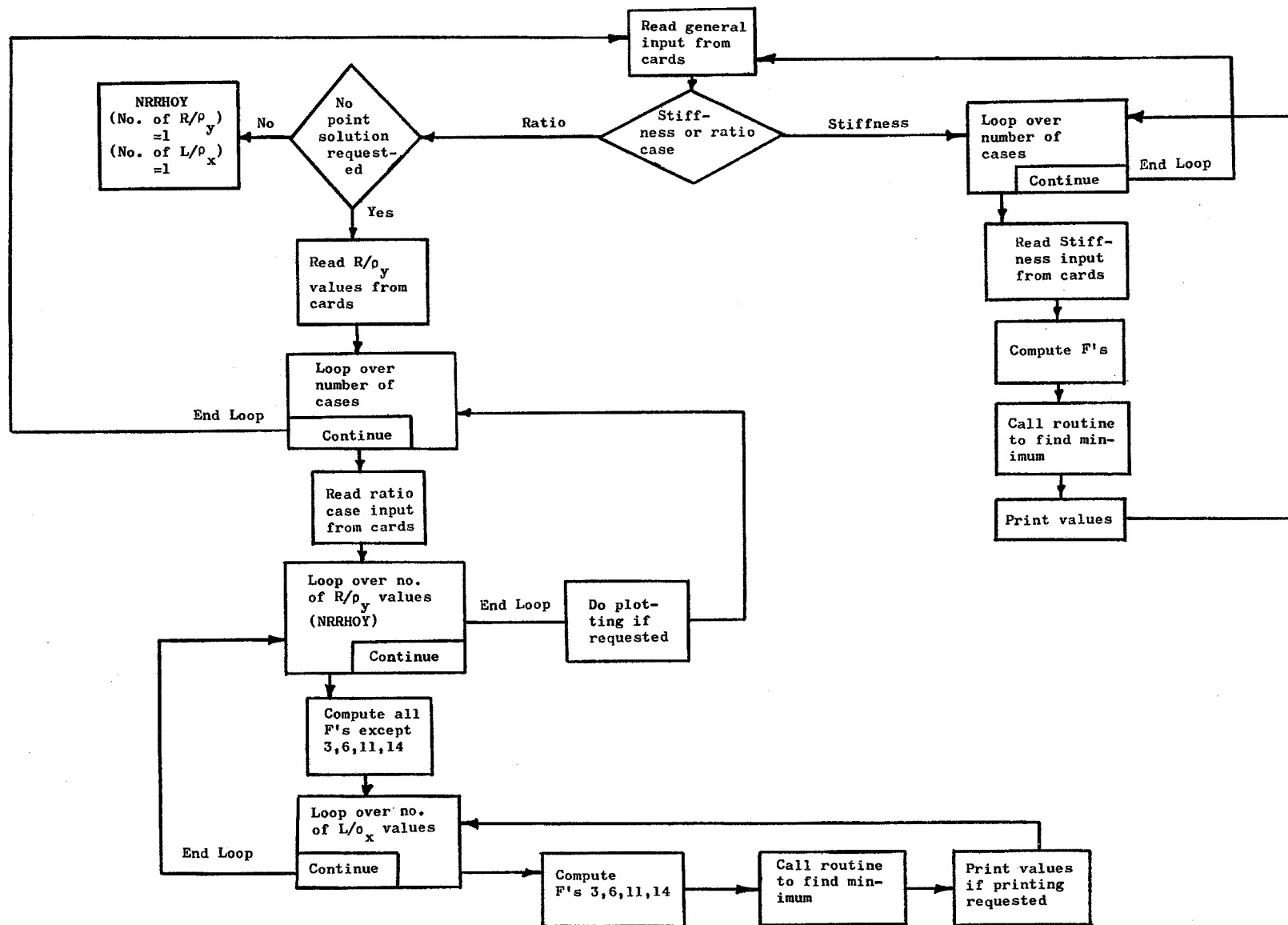


Figure 11 - Flow-Diagram - Program 4267

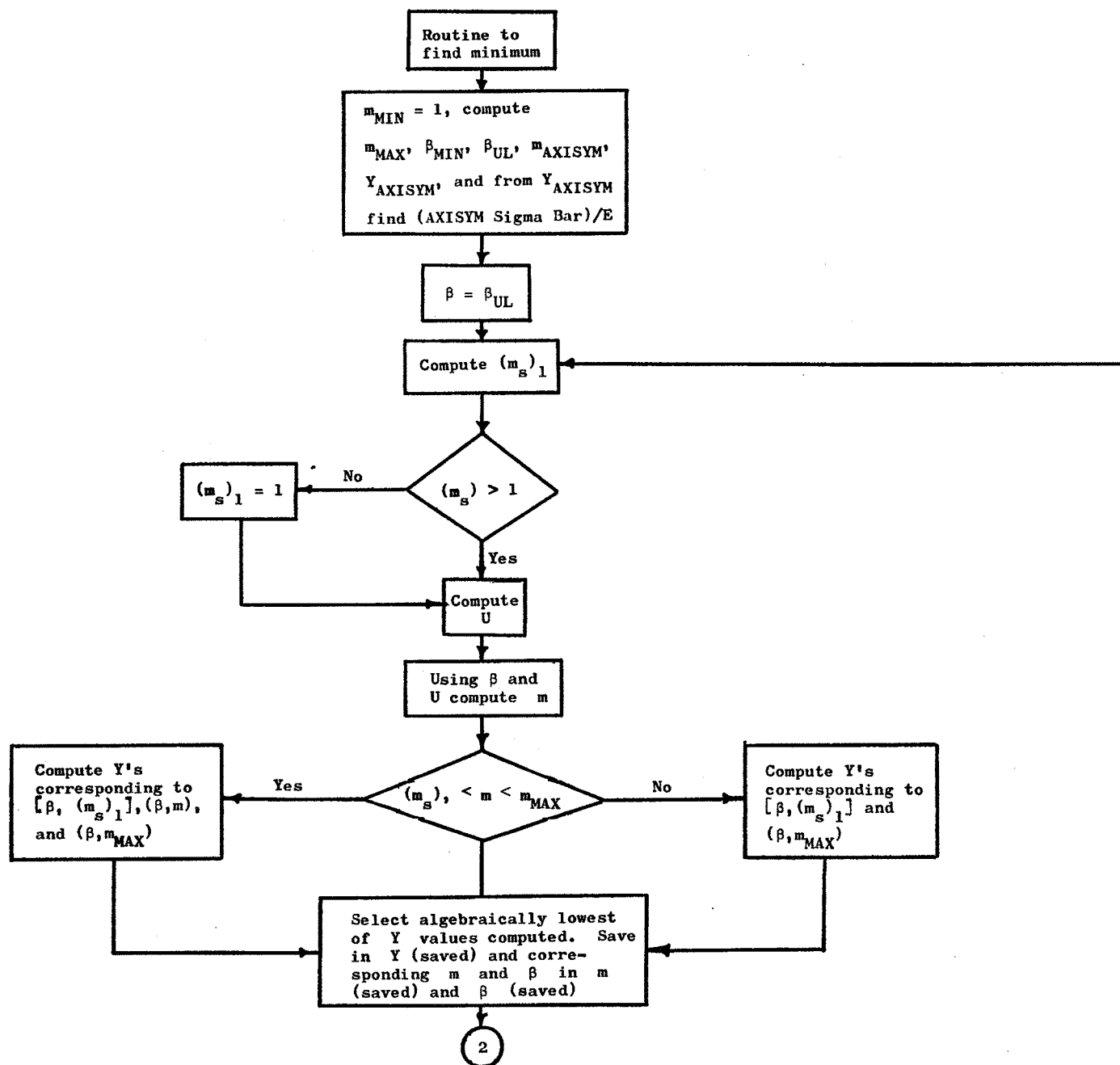


Figure 11 - Flow-Diagram - Program 4267
(Continued)

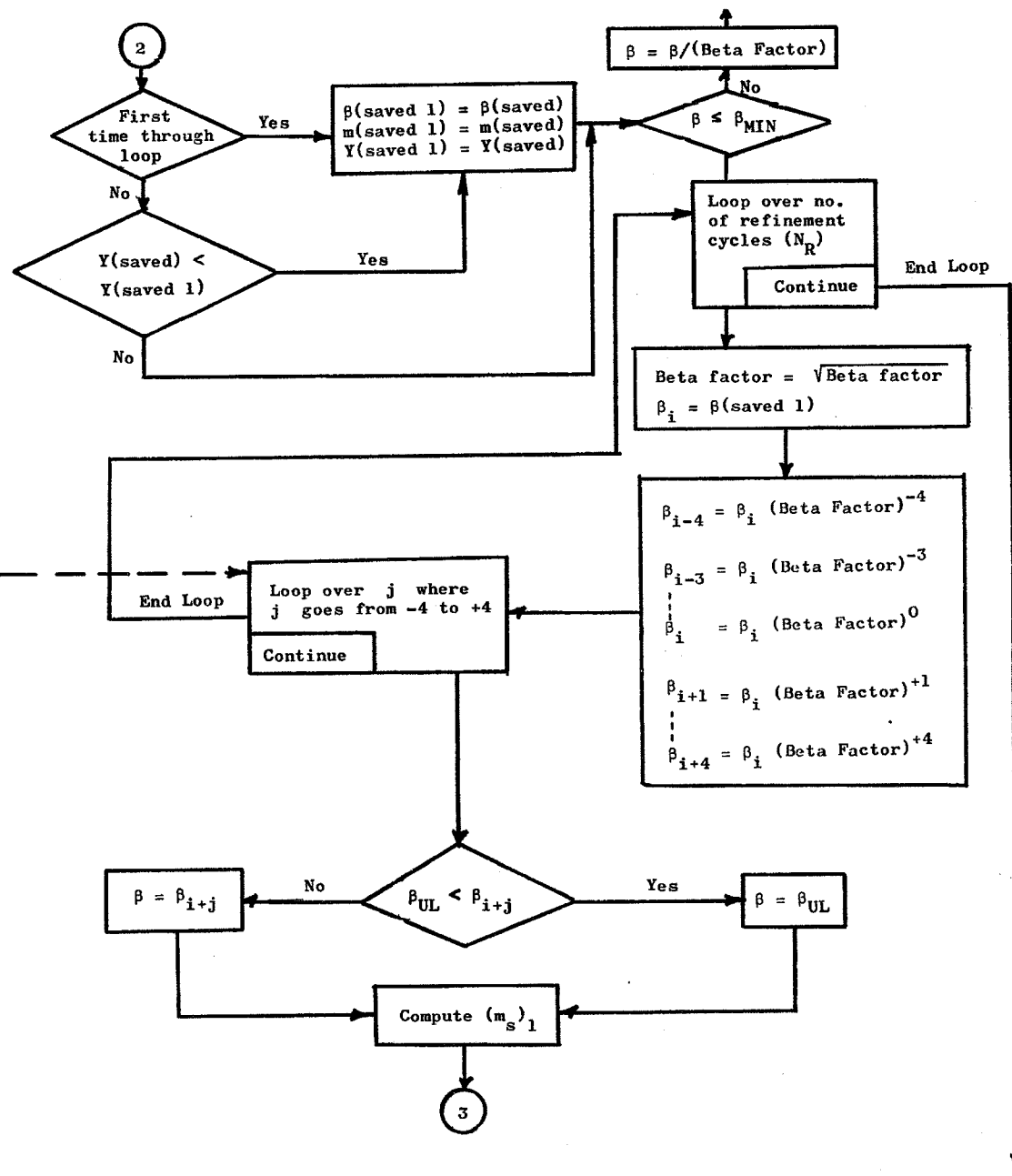
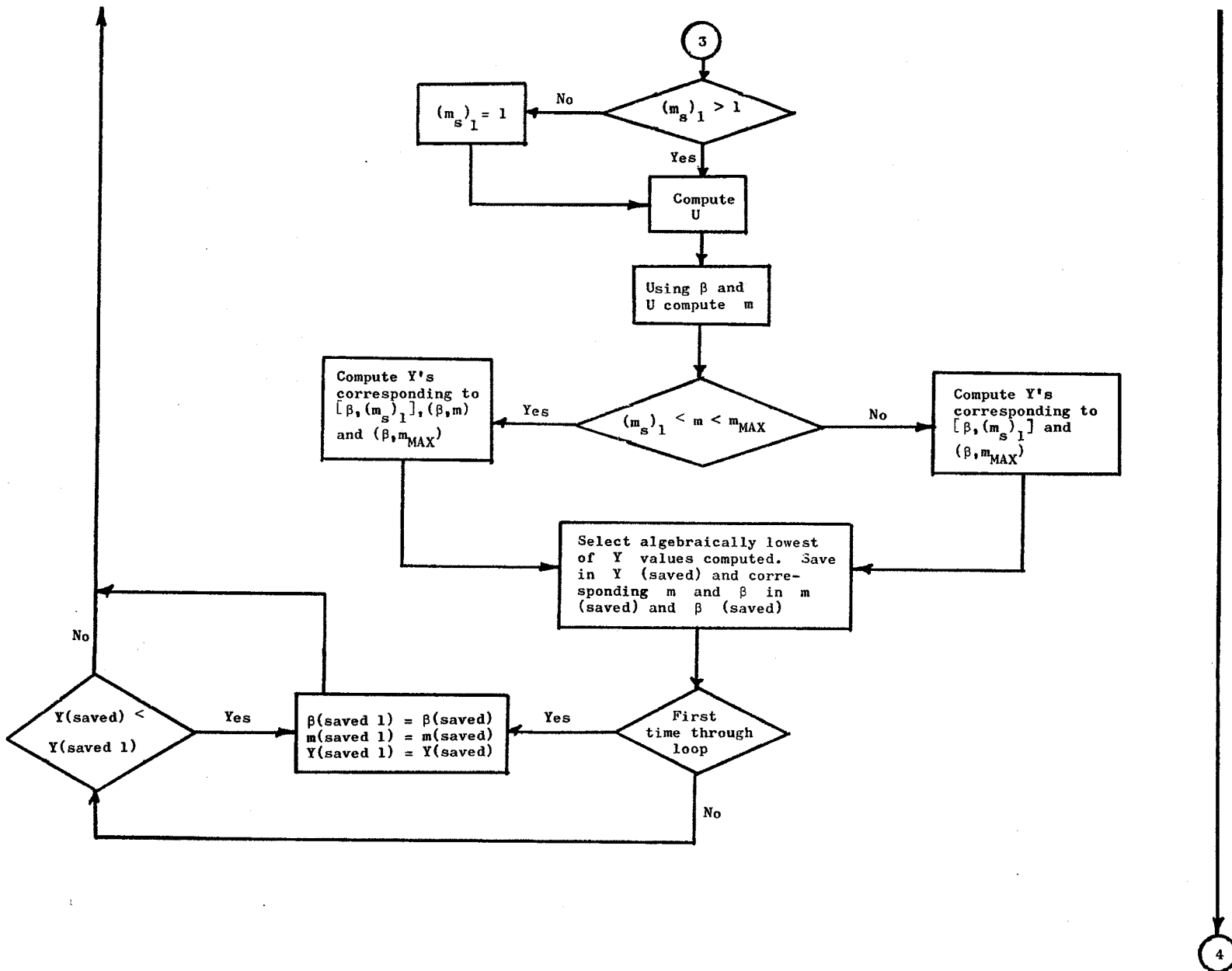


Figure 11 - Flow-Diagram - Program 4267
(Continued)



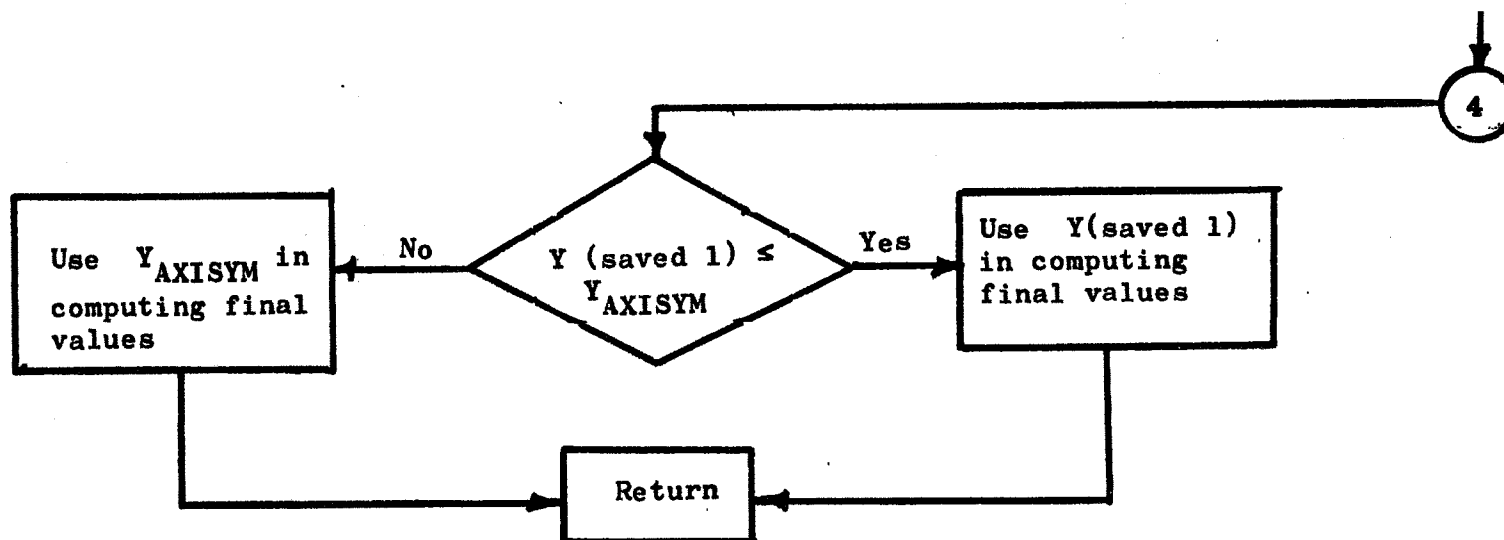


Figure 11 - Flow-Diagram - Program 4267
(Continued)

TABLE V - Fortran Listing - Program 4267

```

SIBFTC READD  M94/2,LIST
COMMON/CSTRAT/DUM(50)
C*****
C  STANDARD INPUT FOR ALL OPTIONS
COMMON/ALL/
C
C          PROBLEM IDENTIFICATION
C  * TITLE(10)
C          INPUT OPTION (RATIO OR STIFF)
C  *,RAORST
C          NUMBER OF CASES
C  *,NCASES
C          POISSON'S RATIO
C  *,POISR
C          E/SIGMA
C  *,EDSIG
C          LOWER C SUB ALPHA
C  *,CALPHL
C          UPPER C SUB ALPHA
C  *,CALPHU
C          LOWER C SUB BETA
C  *,CBETAL
C          UPPER C SUB BETA
C  *,CBETAU
C          THE NUMBER OF (R/RHO SUB Y) RATIOS
C          TO BE INCLUDED IN PLOTS OR TABLES
C          WHICH RESULT FROM AUTOMATIC SEQUE
C          NCING OPERATIONS. WILL BE LEFT BL
C          ANK WHEN ONLY POINT SOLUTIONS ARE
C          TO BE OBTAINED
C  *,NRRHOY
C          ETA SUB P OPTION (BLANK OR 1)
C  *,IETAP
C          C SUB 12 OPTION  (BLANK OR 1)
C  *,IC12
C          C SUB 21 OPTION  (BLANK OR 1)
C  *,IC21
C          A SUB 12 OPTION  (BLANK OR 1)
C  *,IA12
C          N SUB R *** THE NUMBER OF REFINE-
C          MENT CYCLES TO BE EMPLOYED IN THE
C          MINIMIZATION PROCEDURE
C  *,NR
C          DUMP OPTION *** 1 WHEN SUPPLEMENT
C          ARY DIAGNOSTIC OUTPUT DESIRED
C  *,IDUMP
C          MAXIMUM ORDINATE FOR PLOTS
C  *,ORDMAX
C          SCREENING CUT OFF FOR THE MINIMIZ
C          ATION PROCESS
C  *,SCREEN
C          MINIMUM NUMBER OF CIRCUMFERENTIAL

```


TABLE V - Fortran Listing - Program 4267
(Continued)

```

C
C
C
      HALF WAVES USED TO ESTABLISH A
      CUT OFF VALUE IN THE MINIMIZATION
      PROCEDURE
      ,DUMP
      *,CHWMIN
      LOGICAL DUMP
      DUMP=,FALSE.
      DATA RATI /5HRATIO/
      *,STIF /5HSTIFF/
      *,BLANK/6H      /
      I5=5
      I6=6
      I8=8
100  CONTINUE
      ASSIGN 1500 TO IEOF
      CALL ERCTRL(34,IEOF)
      CALL ERRTR(-34)
      READ(I5,200)TITLE
200  FORMAT(10A6)
      CALL ERCTRL(34,0)
      CALL ERRTR(34)
      DUMP=,FALSE.
      READ(I5,300)RAORST,NCASES,POISR,EDSIG,CALPHL,CALPHU,CBETAL,CBETAU,
      *NRRHOY,IETAP,IC12,IC21,IA12,NR,IDUMP
300  FORMAT(A5,I5,6(F5.0),7(I5))
      READ(I5,400)ORDMAX,SCREEN,CHWMIN
400  FORMAT(3E10.5)
      IF(IDUMP.EQ.1)DUMP=,TRUE.
      IF(RAORST.EQ.RATI )GO TO 600
      IF(RAORST.EQ.STIF )GO TO 1000
      WRITE(I6,500) RAORST
500  FORMAT(1H0,45HERROR IN OPTION REQUESTED NOT RATIO OR STIFF ,A5)
      RETURN
600  CONTINUE
950  CALL RATIO
      GO TO 100
1000 CONTINUE
      CALL STIFF
      GO TO 100
1500 RETURN
      END

$IBMAP TAPE
      ENTRY      .UN08.
      .UN08. PZE      UNIT08
      UNIT08 FILE      ,A(1),READY,OUTPUT,BLK=115,BCD
      END
$IBFTC RATIOD M94/2,LIST
      SUBROUTINE RATIO
      COMMON/INTER/
      *F1
      *,F2

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

*,F3
*,F4
*,F5
*,F6
*,F7
*,F8
*,F9
*,F10
*,F11
*,F12
*,F13
*,F14
*,F15
*,SOLUT(8)
*,ARRAY(11,25)
*,FLROX(11)
*,FMMAX
C*****
C  STANDARD INPUT FOR ALL OPTIONS
C  COMMON/ALL/
C
C          PROBLEM IDENTIFICATION
C  * TITLE(10)
C
C          INPUT OPTION (RATIO OR STIFF)
C  *,RAORST
C
C          NUMBER OF CASES
C  *,NCASES
C
C          POISSON'S RATIO
C  *,POISR
C
C          E/SIGMA
C  *,EDSIG
C
C          LOWER C SUB ALPHA
C  *,CALPHL
C
C          UPPER C SUB ALPHA
C  *,CALPHU
C
C          LOWER C SUB BETA
C  *,CBETAL
C
C          UPPER C SUB BETA
C  *,CBETAU
C
C          THE NUMBER OF (R/RHO SUB Y) RATIOS
C          TO BE INCLUDED IN PLOTS OR TABLES
C          WHICH RESULT FROM AUTOMATIC SEQUE
C          NCING OPERATIONS. WILL BE LEFT BL
C          ANK WHEN ONLY POINT SOLUTIONS ARE
C          TO BE OBTAINED
C  *,NRRHOY
C
C          ETA SUB P OPTION (BLANK OR 1)
C  *,IETAP
C
C          C SUB 12 OPTION  (BLANK OR 1)
C  *,IC12
C
C          C SUB 21 OPTION  (BLANK OR 1)
C  *,IC21

```

TABLE V - Fortran Listing - Program 4267
(Continued)

C		A SUB 12 OPTION (BLANK OR 1)
C	* ,IA12	
C		N SUB R *** THE NUMBER OF REFINEMENT CYCLES TO BE EMPLOYED IN THE MINIMIZATION PROCEDURE
C		
C	* ,NR	
C		DUMP OPTION *** 1 WHEN SUPPLEMENTARY DIAGNOSTIC OUTPUT DESIRED
C	* ,IDUMP	
C		MAXIMUM ORDINATE FOR PLOTS
C	* ,ORDMAX	
C		SCREENING CUT OFF FOR THE MINIMIZATION PROCESS
C	* ,SCREEN	
C		MINIMUM NUMBER OF CIRCUMFERENTIAL HALF WAVES USED TO ESTABLISH A CUT OFF VALUE IN THE MINIMIZATION PROCEDURE
C		,DUMP
C	* ,CHWMIN	
C	LOGICAL DUMP	
C	*****	
C	RATIO INPUT INFORMATION	
C	COMMON/CSTRAT/	
C		CASE NUMBER
C	* NCASE	
C		OUTPUT OPTION 1=TABLES, 2=TABLES AND PLOTS, 3=PLOTS, 4=POINT SOL.
C	* ,ITYPEO	
C		THICKNESS RATIO (T BAR SUB X/T)
C	* ,THICKX	
C		LENGTH TO RADIUS RATIO (L/R)
C	* ,FLGRAD	
C		THICKNESS RATIO (T BAR SUB Y/T)
C	* ,THICKY	
C		GEOMETRIC FACTOR (ONLY FOR POINT SOLUTION) C SUB RHO X
C	* ,CRHOX	
C		GEOMETRIC FACTOR (ONLY FOR POINT SOLUTION) C SUB H X
C	* ,CHX	
C		GEOMETRIC FACTOR (ONLY FOR POINT SOLUTION) C SUB RHO Y
C	* ,CRHOY	
C		GEOMETRIC FACTOR (ONLY FOR POINT SOLUTION) C SUB H Y
C	* ,CHY	
C		STRINGER LOCATION (INSIDE,OUTSIDE ,SYMMETRIC
C	* ,STRING(2)	
C		RING LOCATION (INSIDE,OUTSIDE,SYMMETRIC)

TABLE V - Fortran Listing - Program 4267
(Continued)

C	* ,RINGL(2)	(L/RHO SUB X) LONGITUDINAL SLENDE
C		RNESS RATIO (POINT SOL. ONLY)
C	* ,FLRHOX	(R/RHO SUB Y) CIRCUMFERENTIAL SLE
C		NDERNESS RATIO (POINT SOL. ONLY)
C	* ,FRRHOY	INCREMENTING FACTOR USED IN MINI-
C		MIZATION PROCESS
C	* ,BETAF	(L/RHO SUB X Y) ARTIFICIAL SLENDE
C		RNESS RATIO (POINT SOL. ONLY)
C	* ,FLROXY	(C11/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	* ,C11R	(C22/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	* ,C22R	(C12/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	* ,C12R	(C21/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C		R/RHO SUB Y RATIOS TO BE IN PLOTS
C		OR TABLES
	* ,C21R	,RDRHOY(25)
	DIMENSION CH2(2)	
	* ,AUTO(2)	
	LOGICAL POINT	
	DATA(AUTO(I),I=1,2)/6HAUTOMA,3HTIC/	
	* ,FINSID/6HINSIDE/	
	* ,OUTSID/6HOUTSID/	
	* ,SYMMET/6HSYMMET/	
	POINT=.FALSE.	
	PI=3.1415926	
	PI2=PI*PI	
	ICOUNT=0	
	IFLAG=0	
	I5=5	
	I6=6	
	I8=8	
	FLROX(2)=42.5	
	FLROX(3)=60.0	
	FLROX(4)=86.0	
	FLROX(5)=121.5	
	FLROX(6)=175.0	
	FLROX(7)=245.0	
	FLROX(8)=350.0	
	FLROX(9)=495.0	
	FLROX(10)=700.0	

TABLE V - Fortran Listing - Program 4267
(Continued)

```

FLROX(11)=1000.0
100 CONTINUE
    IEND=11
    FLROX(1)=30.0
    IF(NRRHOY.EQ.0)GO TO 200
    READ(I5,150)(RDRHOY(I),I=1,NRRHOY)
150 FORMAT(8E10.5)
200 CONTINUE
    DO 5000 JJ=1,NCASES
    READ(I5,250)NCASE,ITYPE0,THICKX,FLGRAD,THICKY,CRHOX,CH2,CRHOY,CHY
250 FORMAT(I5,I5,4E10.5,A6,A4,2E10.5)
    ICOUNT=ICOUNT+1
    DO 300 I=1,2
    IF(CH2(I).NE.AUTO(I))GO TO 350
300 CONTINUE
    CHX=(1.0/2.0)*(THICKX-1.0)/THICKX
    GO TO 450
350 READ400,NCASE,ITYPE0,THICKX,FLGRAD,THICKY,CRHOX,CHX,CRHOY,CHY
400 FORMAT(I5,I5,7E10.5)
450 CONTINUE
    READ(I5,500)STRING,RINGL,BETAF
500 FORMAT(A6,A4,A6,A4,E10.5)
    IF(ITYPE0.NE.4)GO TO 1000
    READ(I5,550)FLRHOX,FRRHOY,FLROXY,C11R,C22R,C12R,C21R
550 FORMAT(7E10.5)
    RDRHOY=FRRHOY
900 POINT=.TRUE.
1000 CONTINUE
    IF(ITYPE0.EQ.3) GO TO 1050
    CALL PRINT(IFLAG)
1050 CONTINUE
    DO 2000 I=1,NRRHOY
    F1=PI2/(RDRHOY(I)*RDRHOY(I)*FLGRAD*FLGRAD)
    IF(DUMP)WRITE(I8,10000)F1
    F2=0.0
    IF(POINT)F2=(2.0*PI2)/(EDSIG*(THICKX**.5)*(THICKY**.5)*FLROXY*FLRO
*XY)
    IF(IETAP.EQ.0)F2=0.0
    IF(DUMP)WRITE(I8,10100)F2
    F4=(4.0*PI2)/(FLGRAD*FLGRAD)
    IF(DUMP)WRITE(I8,10300)F4
    F5=(1.0/(2.0*PI2))*FLGRAD*FLGRAD*(THICKY**.5/THICKX**.5)
    IF(DUMP)WRITE(I8,10400)F5
    IF(STRING.EQ.FINSID)FK3X=1.0
    IF(POINT) GO TO 1100
    IF(STRING.EQ.OUTSID)FK3X=-1.0
    IF(STRING.EQ.SYMMET)FK3X=0.0
    IF(RINGL.EQ.FINSID)FK3Y=1.0
    IF(RINGL.EQ.OUTSID)FK3Y=-1.0
    IF(RINGL.EQ.SYMMET)FK3Y=0.0
    C22R=(FK3Y*CHY)/(CRHOY*RDRHOY(I))

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

C12R=0.0
C21R=0.0
1100 CONTINUE
F7=(.5*C12R*(THICKY**.5/THICKX**.5))
IF(IC12.EQ.0)F7=0.0
IF(DUMP)WRITE(I8,10600)F7
F8=(.5*C21R*(THICKX**.5/THICKY**.5))
IF(IC21.EQ.0)F8=0.0
IF(DUMP)WRITE(I8,10700)F8
F9=(-1.0)*POISR*((THICKY**.5)/(THICKX**.5))
IF(IA12.EQ.0)F9=0.0
IF(DUMP)WRITE(I8,10800)F9
F10=EDSIG*(THICKX**.5)*(THICKY**.5)
IF(DUMP)WRITE(I8,10900)F10
F12=(1.0/PI2)*FLGRAD*FLGRAD*(THICKY/THICKX)
IF(DUMP)WRITE(I8,11100)F12
F13=PI2*(C12R/(FLGRAD*FLGRAD))
IF(IC12.EQ.0)F13=0.0
IF(DUMP)WRITE(I8,11200)F13
F15=((PI2*PI2)/(FLGRAD**4.0))*C12R*C12R
IF(DUMP)WRITE(I8,11400)F15
IF(.NOT.POINT)GO TO 1200
IEND=1
FLROX(1)=FLRHOX
1200 DO 1900 J=1,IEND
F3=PI2/(FLROX(J)*FLROX(J))
IF(DUMP)WRITE(I8,10200)F3
IF(.NOT.POINT)C11R=((FK3X*CHX)/CRHOX)*FLGRAD*(1.0/FLROX(J))
F6=.5*(C11R+C22R)
IF(DUMP)WRITE(I8,10500)F6
F11=PI2/(FLROX(J)*FLROX(J))
IF(DUMP)WRITE(I8,11000)F11
F14=(PI2*PI2*THICKX)/(FLGRAD*FLGRAD*THICKY*FLROX(J)*FLROX(J))
IF(DUMP)WRITE(I8,11300)F14
IOPT=0
CALL MINMUM(IOPT)
IF(ITYPE0.EQ.2.OR.ITYPE0.EQ.3)ARRAY(J,I)=SOLUT(3)
IF(ITYPE0.EQ.3) GO TO 1900
CALL PRINT2(J,I)
1900 CONTINUE
2000 CONTINUE
IF(ITYPE0.EQ.2.OR.ITYPE0.EQ.3)CALL PLOT
5000 CONTINUE
RETURN
10000 FORMAT(1H0,10X,6HF1  = ,1PE12.5)
10100 FORMAT(1H0,10X,6HF2  = ,1PE12.5)
10200 FORMAT(1H0,10X,6HF3  = ,1PE12.5)
10300 FORMAT(1H0,10X,6HF4  = ,1PE12.5)
10400 FORMAT(1H0,10X,6HF5  = ,1PE12.5)
10500 FORMAT(1H0,10X,6HF6  = ,1PE12.5)
10600 FORMAT(1H0,10X,6HF7  = ,1PE12.5)

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

10700 FORMAT(1H0,10X,6HF8  = ,1PE12.5)
10800 FORMAT(1H0,10X,6HF9  = ,1PE12.5)
10900 FORMAT(1H0,10X,6HF10 = ,1PE12.5)
11000 FORMAT(1H0,10X,6HF11 = ,1PE12.5)
11100 FORMAT(1H0,10X,6HF12 = ,1PE12.5)
11200 FORMAT(1H0,10X,6HF13 = ,1PE12.5)
11300 FORMAT(1H0,10X,6HF14 = ,1PE12.5)
11400 FORMAT(1H0,10X,6HF15 = ,1PE12.5)
      END

```

```

$IBFTC STIFFD  M94/2,LIST
      SUBROUTINE STIFF

```

```

C*****

```

```

C      INPUT FOR STIFF OPTION
C      COMMON/CSTRAT/

```

C		CASE NUMBER
C	* NCASE	
C		ELASTIC CONSTANT
C	* ,A11	
C		ELASTIC CONSTANT
C	* ,A22	
C		ELASTIC CONSTANT
C	* ,A12	
C		ELASTIC CONSTANT
C	* ,A33	
C		ECCENTRICITY COUPLING CONSTANT
C	* ,C11	
C		ECCENTRICITY COUPLING CONSTANT
C	* ,C22	
C		RADIUS TO MIDDLE SURFACE OF BASIC
C		CYLINDRICAL SKIN
C	* ,R	
C		ELASTIC CONSTANT
C	* ,D11	
C		ELASTIC CONSTANT
C	* ,D22	
C		ELASTIC CONSTANT
C	* ,D12	
C		ELASTIC CONSTANT
C	* ,D33	
C		ECCENTRICITY COUPLING CONSTANT
C	* ,C12	
C		ECCENTRICITY COUPLING CONSTANT
C	* ,C21	
C		OVERALL LENGTH OF CYLINDER
C	* ,FLENGT	
C		AN INCREMENTING FACTOR USED IN MI
C		NIMIZATION PROCESS
	* ,BETAF	
	COMMON/INTER/	
	* F1	

TABLE V - Fortran Listing - Program 4267
(Continued)

```

*,F2
*,F3
*,F4
*,F5
*,F6
*,F7
*,F8
*,F9
*,F10
*,F11
*,F12
*,F13
*,F14
*,F15
*,SOLUT(8)
*,ARRAY(11,25)
*,FLROX(11)
*,FM MAX
C*****
C  STANDARD INPUT FOR ALL OPTIONS
C  COMMON/ALL/
C
C          PROBLEM IDENTIFICATION
C  * TITLE(10)
C
C          INPUT OPTION (RATIO OR STIFF)
C  *,RAORST
C
C          NUMBER OF CASES
C  *,NCASES
C
C          POISSON'S RATIO
C  *,POISR
C
C          E/SIGMA
C  *,EDSIG
C
C          LOWER C SUB ALPHA
C  *,CALPHL
C
C          UPPER C SUB ALPHA
C  *,CALPHU
C
C          LOWER C SUB BETA
C  *,CBETAL
C
C          UPPER C SUB BETA
C  *,CBETAU
C
C          THE NUMBER OF (R/RHO SUB Y) RATIOS
C          TO BE INCLUDED IN PLOTS OR TABLES
C          WHICH RESULT FROM AUTOMATIC SEQUE
C          NCING OPERATIONS. WILL BE LEFT BL
C          ANK WHEN ONLY POINT SOLUTIONS ARE
C          TO BE OBTAINED
C  *,NRRHOY
C
C          ETA SUB P OPTION (BLANK OR 1)
C  *,IETAP
C
C          C SUB 12 OPTION  (BLANK OR 1)
C  *,IC12
C          C SUB 21 OPTION  (BLANK OR 1)

```


TABLE V - Fortran Listing - Program 4267
(Continued)

C	* ,IC21	A SUB 12 OPTION (BLANK OR 1)
C C C	* ,IA12	N SUB R *** THE NUMBER OF REFINEMENT CYCLES TO BE EMPLOYED IN THE MINIMIZATION PROCEDURE
C C	* ,NR	DUMP OPTION *** 1 WHEN SUPPLEMENTARY DIAGNOSTIC OUTPUT DESIRED
C	* ,IDUMP	MAXIMUM ORDINATE FOR PLOTS
C C	* ,ORDMAX	SCREENING CUT OFF FOR THE MINIMIZATION PROCESS
C C C C	* ,SCREEN	MINIMUM NUMBER OF CIRCUMFERENTIAL HALF WAVES USED TO ESTABLISH A CUT OFF VALUE IN THE MINIMIZATION PROCEDURE
	* ,CHWMIN	,DUMP
	LOGICAL DUMP	
	I5=5	
	I6=6	
	I8=8	
	IFLAG=0	
100	CONTINUE	
	DO 1000 K=1,NCASES	
	READ(I5,150)NCASE,A11,A22,A12,A33,C11,C22,R	
150	FORMAT(I10,7E10.5)	
	READ(I5,200)D11,D22,D12,D33,C12,C21,FLENGT	
200	FORMAT(10X,7E10.5)	
	READ(I5,250)BETAF	
250	FORMAT(10X,E10.5)	
	PI=3.1415926	
	FLENG2=FLENGT*FLENGT	
	PI2=PI*PI	
	R2=R*R	
	F1=(PI2*D22*A22)/FLENG2	
	IF(IC12.EQ.0)F1=0.0	
	IF(DUMP)WRITE(I8,10000)F1	
	F2=(2.0*PI2*(D12+2.0*D33)*((A11*A22)**.5))/FLENG2	
	IF(IETAP.EQ.0)F2=0.0	
	IF(DUMP)WRITE(I8,10100)F2	
	F3=(PI2*D11*A11)/FLENG2	
	IF(DUMP)WRITE(I8,10200)F3	
	F4=(4.0*PI2)/(FLENG2/R2)	
	IF(DUMP)WRITE(I8,10300)F4	
	F5=(1.0/(2.0*PI2))*(FLENG2/R2)*((A11/A22)**.5)	
	IF(DUMP)WRITE(I8,10400)F5	
	F6=(1.0/2.0)*((C11/R)+(C22/R))	
	IF(DUMP)WRITE(I8,10500)F6	

TABLE V - Fortran Listing - Program 4267
(Continued)

```

F7=(1.0/2.0)*(C12/R)*((A11/A22)**.5)
IF(IC12.EQ.0)F7=0.0
IF(DUMP)WRITE(I8,10600)F7
F8=(1.0/2.0)*(C21/R)*((A22/A11)**.5)
IF(IC21.EQ.0)F8=0.0
IF(DUMP)WRITE(I8,10700)F8
F9=(2.0*A12)/((A11*A22)**.5)
IF(1A12.EQ.0)F9=0.0
IF(DUMP)WRITE(I8,10800)F9
F10=A33/((A11*A22)**.5)
IF(DUMP)WRITE(I8,10900)F10
F11=(PI2*D11*A11)/FLENG2
IF(DUMP)WRITE(I8,11000)F11
F12=(1.0/F12)*(FLENG2/R2)*(A11/A22)
IF(DUMP)WRITE(I8,11100)F12
F13=(PI2*C12*R)/FLENG2
IF(DUMP)WRITE(I8,11200)F13
F14=(PI2*PI2*R2*D11*A22)/(FLENG2*FLENG2)
IF(DUMP)WRITE(I8,11300)F14
F15=(PI2*PI2*R2*C12*C12)/(FLENG2*FLENG2)
IF(DUMP)WRITE(I8,11400)F15
IOPT=1
CALL MINMUM(IOPT)
CALL PRINT3(K)
1000 CONTINUE
      RETURN
10000 FORMAT(1H0,10X,6HF1  = ,1PE12.5)
10100 FORMAT(1H0,10X,6HF2  = ,1PE12.5)
10200 FORMAT(1H0,10X,6HF3  = ,1PE12.5)
10300 FORMAT(1H0,10X,6HF4  = ,1PE12.5)
10400 FORMAT(1H0,10X,6HF5  = ,1PE12.5)
10500 FORMAT(1H0,10X,6HF6  = ,1PE12.5)
10600 FORMAT(1H0,10X,6HF7  = ,1PE12.5)
10700 FORMAT(1H0,10X,6HF8  = ,1PE12.5)
10800 FORMAT(1H0,10X,6HF9  = ,1PE12.5)
10900 FORMAT(1H0,10X,6HF10 = ,1PE12.5)
11000 FORMAT(1H0,10X,6HF11 = ,1PE12.5)
11100 FORMAT(1H0,10X,6HF12 = ,1PE12.5)
11200 FORMAT(1H0,10X,6HF13 = ,1PE12.5)
11300 FORMAT(1H0,10X,6HF14 = ,1PE12.5)
11400 FORMAT(1H0,10X,6HF15 = ,1PE12.5)
      END

$IBFTC MINIMD A94/2,LIST
      SUBROUTINE MINMUM(IOPT)
C*****
C      STANDARD INPUT FOR ALL OPTIONS
      COMMON/ALL/
C
C      PROBLEM IDENTIFICATION
C      * TITLE(10)
C      INPUT OPTION (RATIO OR STIFF)
C      *,RAORST

```

TABLE V - Fortran Listing - Program 4267
(Continued)

C		NUMBER OF CASES
C	* , NCASES	
C		POISSON'S RATIO
C	* , POISR	
C		E/SIGMA
C	* , EDSIG	
C		LOWER C SUB ALPHA
C	* , CALPHL	
C		UPPER C SUB ALPHA
C	* , CALPHU	
C		LOWER C SUB BETA
C	* , CBETAL	
C		UPPER C SUB BETA
C	* , CBETAU	
C		THE NUMBER OF (R/RHO SUB Y) RATIOS
C		TO BE INCLUDED IN PLOTS OR TABLES
C		WHICH RESULT FROM AUTOMATIC SEQUE
C		NCING OPERATIONS. WILL BE LEFT BL
C		ANK WHEN ONLY POINT SOLUTIONS ARE
C		TO BE OBTAINED
C	* , NRRHOY	
C		ETA SUB P OPTION (BLANK OR 1)
C	* , IETAP	
C		C SUB 12 OPTION (BLANK OR 1)
C	* , IC12	
C		C SUB 21 OPTION (BLANK OR 1)
C	* , IC21	
C		A SUB 12 OPTION (BLANK OR 1)
C	* , IA12	
C		N SUB R *** THE NUMBER OF REFINE-
C		MENT CYCLES TO BE EMPLOYED IN THE
C		MINIMIZATION PROCEDURE
C	* , NR	
C		DUMP OPTION *** 1 WHEN SUPPLEMENT
C		ARY DIAGNOSTIC OUTPUT DESIRED
C	* , IDUMP	
C		MAXIMUM ORDINATE FOR PLOTS
C	* , ORDMAX	
C		SCREENING CUT OFF FOR THE MINIMIZ
C		ATION PROCESS
C	* , SCREEN	
C		MINIMUM NUMBER OF CIRCUMFERENTIAL
C		HALF WAVES USED TO ESTABLISH A
C		CUT OFF VALUE IN THE MINIMIZATION
C		PROCEDURE
C	* , CHWMIN	, DUMP
	LOGICAL DUMP	
	COMMON/INTER/	
	* F1	
	* , F2	
	* , F3	

TABLE V - Fortran Listing - Program 4267
(Continued)

```

*,F4
*,F5
*,F6
*,F7
*,F8
*,F9
*,F10
*,F11
*,F12
*,F13
*,F14
*,F15
*,SOLUT(8)
*,ARRAY(11,25)
*,FLROX(11)
*,FMMAX
C*****
C  RATIO INPUT INFORMATION
C  COMMON/CSTRAT/
C
C          CASE NUMBER
C
C          * NCASE
C
C          OUTPUT OPTION 1=TABLES, 2=TABLES
C          AND PLOTS, 3=PLOTS, 4=POINT SOL.
C
C          *,ITYPEO
C
C          THICKNESS RATIO (T BAR SUB X/T)
C
C          *,THICKX
C
C          LENGTH TO RADIUS RATIO (L/R)
C
C          *,FLGRAD
C
C          THICKNESS RATIO (T BAR SUB Y/T)
C
C          *,THICKY
C
C          GEOMETRIC FACTOR (ONLY FOR POINT
C          SOLUTION) C SUB RHO X
C
C          *,CRHOX
C
C          GEOMETRIC FACTOR (ONLY FOR POINT
C          COLUTION) C SUB H X
C
C          *,CHX
C
C          GEOMETRIC FACTOR (ONLY FOR POINT
C          SOLUTION) C SUB RHO Y
C
C          *,CRHOY
C
C          GEOMETRIC FACTOR (ONLY FOR POINT
C          SOLUTION) C SUB H Y
C
C          *,CHY
C
C          STRINGER LOCATION (INSIDE,OUTSIDE
C          ,SYMMETRIC
C
C          *,STRING(2)
C
C          RING LOCATION (INSIDE,OUTSIDE,SYM
C          METRIC)
C
C          *,RINGL(2)
C
C          (L/RHO SUB X) LONGITUDINAL SLENDE
C          RNESS RATIO (POINT SOL. ONLY)
C
C          *,FLRHOX

```

TABLE V - Fortran Listing - Program 4267
(Continued)

C		(R/RHO SUB Y) CIRCUMFERENTIAL SLE
C		NDERNESS RATIO (POINT SOL. ONLY)
C	*FRRHOY	
C		INCREMENTING FACTOR USED IN MINI-
C		MIZATION PROCESS
C	*BETAF	
C		(L/RHO SUB X Y) ARTIFICIAL SLENDE
C		RNESS RATIO (POINT SOL. ONLY)
C	*FLROXY	
C		(C11/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	*C11R	
C		(C22/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	*C22R	
C		(C12/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	*C12R	
C		(C21/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C		R/RHO SUB Y RATIOS TO BE IN PLOTS
C		OR TABLES
C		,RDRHOY(25)
	*C21R	
	EQUIVALENCE	
	*(ITYPE0,A11)	
	*(THICKX,A22)	
	*(FLGRAD,A12)	
	*(THICKY,A33)	
	*(CRHOX,C11)	
	*(CHX,C22)	
	*(CRHOY,R)	
	*(CHY,D11)	
	*(STRING(1),D22)	
	*(STRING(2),D12)	
	*(RINGL(1),D33)	
	*(RINGL(2),C12)	
	*(FLRHOX,C21)	
	*(FRRHOY,FLENGT)	
	DIMENSION BET(9),YMP(9),FMMP(9)	
	I8=8	
	KOUNT=0	
	PI=3.1415926	
	FMMIN=1.0	
	IF(DUMP)WRITE(I8,10000)FMMIN	
	FMMAX=(SCREEN/F3)**.5	
	IF(FMMAX.LT.2.0)FMMAX=2.0	
	IF(DUMP)WRITE(I8,10100)FMMAX	
	BETMIN=(F1/SCREEN)**.25	
	IF(DUMP)WRITE(I8,10200)BETMIN	
	IF(IOPT)300,300,400	
300	BETAUL=(FMMAX*(THICKX**.25))/(FLGRAD*.5*CHWMIN*(THICKY**.25))*PI	

TABLE V - Fortran Listing - Program 4267
(Continued)

```

      GO TO 450
400  BETAUL=(FMAX*PI*R*(A22**.25))/(FLENGT*.5*CHWMIN*(A11** .25))
450  FMAXSY=(1.0/(F14+F15))**.25
      IF(DUMP)WRITE(I8,10300)BETAUL
      IF(FMAXSY.LT.1.0)FMAXSY=1.0
      IF(DUMP)WRITE(I8,10400)FMAXSY
      YAXSYM=F11*FMAXSY*FMAXSY+(F12/(FMAXSY*FMAXSY))*((1.0-F13*FMAXSY*FM
      *AXSY)**2.0)
      IF(DUMP)WRITE(I8,10500)YAXSYM
      IF(IOPT)500,500,600
500  SOLUT(8)=YAXSYM
      GO TO 700
600  SOLUT(8)=YAXSYM/A11
700  CONTINUE
      BETA=BETAUL
1100 BETA2=BETA*BETA
      BETA4=BETA2*BETA2
      IF(DUMP)WRITE(I8,10600)BETA
      BETAS=BETA
      CALL SEVELE(BETAS,YS,FMS,IOPT)
      KOUNT=KOUNT+1
      IF(KOUNT.LE.1)GO TO 1650
      IF(YS.GE.YS1) GO TO 1700
1650 YS1=YS
      IF(DUMP)WRITE(I8,11000)YS1
      FMSI=FMS
      IF(DUMP)WRITE(I8,11100)FMSI
      BETAS1=BETAS
      IF(DUMP)WRITE(I8,11200)BETAS1
1700 IF(BETAS .LE.BETMIN)GO TO 1800
      BETA=BETA/BETAF
      GO TO 1100
1800 CONTINUE
      BETAF1=BETAF
      DO 2100 I=1,NR
      BETAF2=SQRT(BETAF1)
      IF(DUMP)WRITE(I8,11300)BETAF2
      DO 1900 J=1,9
      FJ=-5+J
      BET(J)=BETAS1/BETAF2**FJ
      IF(BET(J).LE.BETAUL)GO TO 1850
      BET(J)=BETAUL
1850 CALL SEVELE(BET(J),YMP(J),FMMP(J),IOPT)
1900 CONTINUE
      K=1
      DO 2000 J=1,9
      IF(YMP(J).LT.YMP(K))K=J
2000 CONTINUE
      YS1=YMP(K)
      IF(DUMP)WRITE(I8,11400)YS1
      BETAS1=BET(K)

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

      IF(DUMP)WRITE(I8,11500)BETAS1
      FMSI=FMMP(K)
      IF(DUMP)WRITE(I8,11600)FMSI
      BETAF1=BETAF2
2100  CONTINUE
      IF(IOPT)2200,2200,2500
2200  SOLUT(3)=YS1
      SOLUT(1)=FMSI
      SOLUT(2)=(FMSI*PI*(THICKX**,25))/(BETAS1*FLGRAD*(THICKY**,25))
      IF(YS1.GT.YAXSYM)GO TO 2300
      GO TO 2400
2300  SOLUT(3)=YAXSYM
      SOLUT(1)=FMAXSY
      SOLUT(2)=0.0
2400  GO TO 3000
2500  CONTINUE
      IF(YS1.GT.YAXSYM)GO TO 2800
      SOLUT(3)=YS1/A11
      SOLUT(1)=FMSI
      SOLUT(2)=(FMSI*PI*R*(A22**,25))/(BETAS1*FLENGT*(A11**,25))
      GO TO 3000
2800  SOLUT(3)=YAXSYM/A11
      SOLUT(1)=FMAXSY
      SOLUT(2)=0.0
3000  IF(YS1.GT.YAXSYM)GO TO 3500
      FML=CALPHL*FMSI
      FMH=CALPHU*FMSI
      BETL=CBETAL*BETAS1
      BETH=CBETAU*BETAS1
      SOLUT(4)=FCN(BETL,FML)
      SOLUT(5)=FCN(BETH,FML)
      SOLUT(6)=FCN(BETL,FMH)
      SOLUT(7)=FCN(BETH,FMH)
      IF(IOPT)3500,3500,3100
3100  DO 3400 I=4,7
      SOLUT(I)=SOLUT(I)/A11
3400  CONTINUE
3500  RETURN
10000 FORMAT(1H0,10X,12HM SUB MIN = ,1PE12.5)
10100 FORMAT(1H0,10X,12HM SUB MAX = ,1PE12.5)
10200 FORMAT(1H0,10X,15HBETA SUB MIN = ,1PE12.5)
10300 FORMAT(1H0,10X,14HBETA SUB UL = ,1PE12.5)
10400 FORMAT(1H0,10X,15HM SUB AXISYM = ,1PE12.5)
10500 FORMAT(1H0,10X,15HY SUB AXISYM = ,1PE12.5)
10600 FORMAT(1H0,10X, 7HBETA = ,1PE12.5)
11000 FORMAT(1H0,10X,20HITERATIVE Y SAVED = ,1PE12.5)
11100 FORMAT(1H0,10X,20HITERATIVE M SAVED = ,1PE12.5)
11200 FORMAT(1H0,10X,23HITERATIVE BETA SAVED = ,1PE12.5)
11300 FORMAT(1H0,10X,20HBETA FACTOR SUB 2 = ,1PE12.5)
11400 FORMAT(1H0,10X,31HREFINEMENT ITERATION SAVED Y = ,1PE12.5)
11500 FORMAT(1H0,10X,34HREFINEMENT ITERATION SAVED BETA = ,1PE12.5)

```

TABLE V - Fortran Listing - Program 4267
(Continued)

11600 FORMAT(1H0,10X,31HREFINEMENT ITERATION SAVED M = ,1PE12.5)
END

\$1BFTC SEVELD M94/2,LIST

SUBROUTINE SEVELE(BETAS,YS,FMS,IOPT)

COMMON/INTER/

*F1

*F2

*F3

*F4

*F5

*F6

*F7

*F8

*F9

*F10

*F11

*F12

*F13

*F14

*F15

*SOLUT(8)

*ARKAY(11,25)

*FLROX(11)

*FMAX

C*****

C RATIO INPUT INFORMATION
COMMON/CSIRAT/

C CASE NUMBER

* NCASE

OUTPUT OPTION 1=TABLES, 2=TABLES
AND PLOTS, 3=PLOTS, 4=POINT SOL.

* ITYPEO

THICKNESS RATIO (T BAR SUB X/T)

* THICKX

LENGTH TO RADIUS RATIO (L/R)

* FLGRAD

THICKNESS RATIO (T BAR SUB Y/T)

* THICKY

GEOMETRIC FACTOR (ONLY FOR POINT
SOLUTION) C SUB RHO X

* CRHUX

GEOMETRIC FACTOR (ONLY FOR POINT
COLUTION) C SUB H X

* CHX

GEOMETRIC FACTOR (ONLY FOR POINT
SOLUTION) C SUB RHO Y

* CRHOY

GEOMETRIC FACTOR (ONLY FOR POINT
SOLUTION) C SUB H Y

* CHY

TABLE V - Fortran Listing - Program 4267
(Continued)

C		STRINGER LOCATION (INSIDE,OUTSIDE
C		,SYMMETRIC
	*STRING(2)	
C		RING LOCATION (INSIDE,OUTSIDE,SYM
C		METRIC)
	*RINGL(2)	
C		(L/RHO SUB X) LONGITUDINAL SLENDE
C		RNESS RATIO (POINT SOL. ONLY)
	*FLRHOX	
C		(R/RHO SUB Y) CIRCUMFERENTIAL SLE
C		NDERNESS RATIO (POINT SOL. ONLY)
	*FRRHOY	
C		INCREMENTING FACTOR USED IN MINI-
C		MIZATION PROCESS
	*BETAF	
C		(L/RHO SUB X Y) ARTIFICIAL SLENDE
C		RNESS RATIO (POINT SOL. ONLY)
	*FLROXY	
C		(C11/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
	*C11R	
C		(C22/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
	*C22R	
C		(C12/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
	*C12R	
C		(C21/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C		R/RHO SUB Y RATIOS TO BE IN PLOTS
C		OR TABLES
	*C21R	,RDRHOY(25)
C	*****	
C	STANDARD INPUT FOR ALL OPTIONS	
	COMMON/ALL/	
C		PROBLEM IDENTIFICATION
	* TITLE(10)	
C		INPUT OPTION (RATIO OR STIFF)
	*RAORST	
C		NUMBER OF CASES
	*NCASES	
C		POISSON'S RATIO
	*POISR	
C		E/SIGMA
	*EDSIG	
C		LOWER C SUB ALPHA
	*CALPHL	
C		UPPER C SUB ALPHA
	*CALPHU	
C		LOWER C SUB BETA
	*CBETAL	

TABLE V - Fortran Listing - Program 4267
(Continued)

C		UPPER C SUB BETA
C	*CBETAU	THE NUMBER OF (R/RHO SUB Y) RATIOS
C		TO BE INCLUDED IN PLOTS OR TABLES
C		WHICH RESULT FROM AUTOMATIC SEQUE
C		NCING OPERATIONS. WILL BE LEFT BL
C		ANK WHEN ONLY POINT SOLUTIONS ARE
C		TO BE OBTAINED
C	*NRRHOY	ETA SUB P OPTION (BLANK OR 1)
C	*IETAP	C SUB 12 OPTION (BLANK OR 1)
C	*IC12	C SUB 21 OPTION (BLANK OR 1)
C	*IC21	A SUB 12 OPTION (BLANK OR 1)
C	*IA12	N SUB R *** THE NUMBER OF REFINE-
C		MENT CYCLES TO BE EMPLOYED IN THE
C		MINIMIZATION PROCEDURE
C	*NR	DUMP OPTION *** 1 WHEN SUPPLEMENT
C		ARY DIAGNOSTIC OUTPUT DESIRED
C	*IDUMP	MAXIMUM ORDINATE FOR PLOTS
C	*ORDMAX	SCREENING CUT OFF FOR THE MINIMIZ
C		ATION PROCESS
C	*SCREEN	MINIMUM NUMBER OF CIRCUMFERENTIAL
C		HALF WAVES USED TO ESTABLISH A
C		CUT OFF VALUE IN THE MINIMIZATION
C		PROCEDURE
C	*CHWMIN	,DUMP
	LOGICAL DUMP	
	EQUIVALENCE	
	*(ITYPEO,A11)	
	*(THICKX,A22)	
	*(FLGRAD,A12)	
	*(THICKY,A33)	
	*(CRHOX,C11)	
	*(CHX,C22)	
	*(CRHOY,R)	
	*(CHY,D11)	
	*(STRING(1),D22)	
	*(STRING(2),D12)	
	*(RINGL(1),D33)	
	*(RINGL(2),C12)	
	*(FLRHOX,C21)	
	*(FRRHOY,FLENGT)	
	PI=3.1415926	

TABLE V- Fortran Listing - Program 4267
(Continued)

```

PI2=PI*PI
I8=8
BETA=BETAS
BETA2=BETA*BETA
BETA4=BETA2*BETA2
IF(IOPT)800,800,900
800 FMS1=(BETA*FLGRAD*CHWMIN*(THICKY**.25))/(PI*2.0*(THICKX**.25))
GO TO 1000
900 FMS1=(BETA*FLENGT*CHWMIN*(A11**.25))/(PI*R*2.0*(A22**.25))
1000 CONTINUE
IF(FMS1.LT.1.0)FMS1=1.0
IF(DUMP)WRITE(I8,10700)FMS1
OMIC=.5*((F4/F1)**.5)*(F6*(1.0/(BETA2      ))+F7+F8*(1.0/(BETA4      )
*))
IF(DUMP)WRITE(I8,10800)OMIC
FM=(( (F5**2)*F4*4.0*BETA4*BETA4)/(4.*F1*((1.0+(F2/F1)*BETA2+(F3/F1
*)*BETA4)*(1.0+F9*BETA2+F10*BETA2+BETA4)+4.0*OMIC*OMIC*BETA4*BETA4)
**).25
IF(DUMP)WRITE(I8,10900)FM
IF(FMS1.LT.FM.AND.FM.LT.FMMAX)GO TO 1200
Y1=FCN(BETA,FMS1)
IF(DUMP)WRITE(I8,11000)Y1
Y3=FCN(BETA,FMMAX)
IF(DUMP)WRITE(I8,11100)Y3
Y2=ABS(Y1)+ABS(Y3)
GO TO 1300
1200 Y1=FCN(BETA,FMS1)
IF(DUMP)WRITE(I8,11000)Y1
Y2=FCN(BETA,FM)
IF(DUMP)WRITE(I8,11200)Y2
Y3=FCN(BETA,FMMAX)
IF(DUMP)WRITE(I8,11100)Y3
1300 CONTINUE
IF(Y1.LT.Y2.AND.Y1.LT.Y3)GO TO 1500
IF(Y2.LT.Y3)GO TO 1400
FMS= FMMAX
BETAS=BETA
YS=Y3
GO TO 1600
1400 FMS=FM
BETAS=BETA
YS=Y2
GO TO 1600
1500 FMS=FMS1
BETAS=BETA
YS=Y1
1600 CONTINUE
IF(DUMP)WRITE(I8,11300)BETAS
IF(DUMP)WRITE(I8,11400)FMS
IF(DUMP)WRITE(I8,11500)YS
RETURN

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

10700 FORMAT(1H0,10X,18H(M SUB S) SUB 1 = ,1PE12.5)
10800 FORMAT(1H0,10X,10HUPSILON = ,1PE12.5)
10900 FORMAT(1H0,10X, 4HM = ,1PE12.5)
11000 FORMAT(1H0,10X,37HY CORRESPONDING TO (M SUB S) SUB 1 = ,1PE12.5)
11100 FORMAT(1H0,10X,31HY CORRESPONDING TO M SUB MAX = ,1PE12.5)
11200 FORMAT(1H0,10X,23HY CORRESPONDING TO M = ,1PE12.5)
11300 FORMAT(1H0,10X,13HSAVED BETA = ,1PE12.5)
11400 FORMAT(1H0,10X,10HSAVED M = ,1PE12.5)
11500 FORMAT(1H0,10X,10HSAVED Y = ,1PE12.5)
      END

```

```

$IBFTC Y      M94/2,LIST
      FUNCTION FCN(BETA,FM)
      COMMON/INTER/
      *F1
      *,F2
      *,F3
      *,F4
      *,F5
      *,F6
      *,F7
      *,F8
      *,F9
      *,F10
      *,F11
      *,F12
      *,F13
      *,F14
      *,F15
      *,SOLUT(8)
      *,ARRAY(11,25)
      *,FLROX(11)
      *,FM MAX
      I8=8
      BETA2=BETA*BETA
      BETA4=BETA2*BETA2
      FCN=F1*((FM*FM)/BETA4)+F2*((FM*FM)/BETA2)+F3*(FM*FM)+F4*FM*FM*BETA
      *4*(F5*(1./(FM*FM))-F6*(1./BETA2)-F7-F8*(1.0/BETA4))*2 /(1.0+F9*B
      *ETA2+F10*BETA2+BETA4)
      RETURN
      END

```

```

$IBFTC PRINTD M94/2,LIST
      SUBROUTINE PRINT(IFLAG)
      COMMON/INTER/
      *F1
      *,F2
      *,F3
      *,F4
      *,F5
      *,F6

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

*,F7
*,F8
*,F9
*,F10
*,F11
*,F12
*,F13
*,F14
*,F15
*,SOLUT(8)
*,ARRAY(11,25)
*,FLROX(11)
*,FMMAX
C*****
C    RATIO INPUT INFORMATION
C    COMMON/CSTRAT/
C
C                                CASE NUMBER
C
C    * NCASE
C                                OUTPUT OPTION 1=TABLES, 2=TABLES
C                                AND PLOTS, 3=PLOTS, 4=POINT SOL.
C
C    *,ITYPEO
C                                THICKNESS RATIO (T BAR SUB X/T)
C
C    *,THICKX
C                                LENGTH TO RADIUS RATIO (L/R)
C
C    *,FLGRAD
C                                THICKNESS RATIO (T BAR SUB Y/T)
C
C    *,THICKY
C                                GEOMETRIC FACTOR (ONLY FOR POINT
C                                SOLUTION) C SUB RHO X
C
C    *,CRHOX
C                                GEOMETRIC FACTOR (ONLY FOR POINT
C                                COLUTION) C SUB H X
C
C    *,CHX
C                                GEOMETRIC FACTOR (ONLY FOR POINT
C                                SOLUTION) C SUB RHO Y
C
C    *,CRHOY
C                                GEOMETRIC FACTOR (ONLY FOR POINT
C                                SOLUTION) C SUB H Y
C
C    *,CHY
C                                STRINGER LOCATION (INSIDE,OUTSIDE
C                                ,SYMMETRIC)
C
C    *,STRING(2)
C                                RING LOCATION (INSIDE,OUTSIDE,SYM
C                                METRIC)
C
C    *,RINGL(2)
C                                (L/RHO SUB X) LONGITUDINAL SLENDE
C                                RNESS RATIO (POINT SOL. ONLY)
C
C    *,FLRHGX
C                                (R/RHO SUB Y) CIRCUMFERENTIAL SLE
C                                NDERNESS RATIO (POINT SOL. ONLY)
C
C    *,FRRHOY

```

TABLE V - Fortran Listing - Program 4267
(Continued)

C		INCREMENTING FACTOR USED IN MINI-
C		MIZATION PROCESS
C	*BETAF	(L/RHO SUB X-Y) ARTIFICIAL SLENDE
C		RNESS RATIO (POINT SOL. ONLY)
C	*FLROXY	(C11/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	*C11R	(C22/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	*C22R	(C12/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C	*C12R	(C21/R) ECCENTRICITY COUPLING RAT
C		IO (POINT SOL. ONLY)
C		R/RHO SUB Y RATIOS TO BE IN PLOTS
C		OR TABLES
C	*C21R	,RDRHOY(25)
C	*****	
C	STANDARD INPUT FOR ALL OPTIONS	
C	COMMON/ALL/	
C		PROBLEM IDENTIFICATION
C	* TITLE(10)	INPUT OPTION (RATIO OR STIFF)
C	*RAORST	NUMBER OF CASES
C	*NCASES	POISSON'S RATIO
C	*POISR	E/SIGMA
C	*EDSIG	LOWER C SUB ALPHA
C	*CALPHL	UPPER C SUB ALPHA
C	*CALPHU	LOWER C SUB BETA
C	*CBETAL	UPPER C SUB BETA
C	*CBETAU	THE NUMBER OF (R/RHO SUB Y) RATIOS
C		TO BE INCLUDED IN PLOTS OR TABLES
C		WHICH RESULT FROM AUTOMATIC SEQUE
C		NCING OPERATIONS. WILL BE LEFT BL
C		ANK WHEN ONLY POINT SOLUTIONS ARE
C		TO BE OBTAINED
C	*NRRHOY	ETA SUB P OPTION (BLANK OR 1)
C	*IETAP	C SUB 12 OPTION (BLANK OR 1)

TABLE V - Fortran Listing - Program 4267
(Continued)

C	* , IC12	C SUB 21 OPTION (BLANK OR 1)
C	* , IC21	A SUB 12 OPTION (BLANK OR 1)
C	* , IA12	N SUB R *** THE NUMBER OF REFINEMENT CYCLES TO BE EMPLOYED IN THE MINIMIZATION PROCEDURE
C	* , NR	DUMP OPTION *** 1 WHEN SUPPLEMENTARY DIAGNOSTIC OUTPUT DESIRED
C	* , IDUMP	MAXIMUM ORDINATE FOR PLOTS
C	* , ORDMAX	SCREENING CUT OFF FOR THE MINIMIZATION PROCESS
C	* , SCREEN	MINIMUM NUMBER OF CIRCUMFERENTIAL HALF WAVES USED TO ESTABLISH A CUT OFF VALUE IN THE MINIMIZATION PROCEDURE
C	* , CHWMIN	, DUMP
	LOGICAL DUMP	
	EQUIVALENCE	
	* (ITYPE0 , A11)	
	* (THICKX , A22)	
	* (FLGRAD , A12)	
	* (THICKY , A33)	
	* (CRHOX , C11)	
	* (CHX , C22)	
	* (CRHOY , R)	
	* (CHY , D11)	
	* (STRING(1) , D22)	
	* (STRING(2) , D12)	
	* (RINGL(1) , D33)	
	* (RINGL(2) , C12)	
	* (FLRHGX , C21)	
	* (FRRHOY , FLENGT)	
	I5=5	
	I6=6	
	I8=8	
	IF (ITYPE0 .EQ. 3) RETURN	
	IFLAG=IFLAG+1	
	IF (IFLAG .NE. 1) GO TO 200	
	WRITE (I6 , 5000)	
	WRITE (I6 , 5100) TITLE	
	WRITE (I6 , 5200)	
	WRITE (I6 , 5300)	
	WRITE (I6 , 5400) RAORST , NCASES , POISR , EDSIG , CALPHL , CALPHU , CBETAL , CBETA	
	* U , NRRHOY , IETAP , IC12 , IC21 , IA12	
	WRITE (I6 , 5500)	

TABLE V - Fortran Listing - Program 4267
(Continued)

```

WRITE(I6,5600)
IF(ITYPEO.EQ.4)GO TO 100
WRITE(I6,5700)ORDMAX,SCREEN,CHWMIN,NR
WRITE(I6,5800)
WRITE(I6,5900)(RDRHOY(I),I=1,NRRHOY)
GO TO 200
100 CONTINUE
WRITE(I6,7500)SCREEN,CHWMIN,NR
200 CONTINUE
IF(IFLAG.NE.1) WRITE(I6,7750)
WRITE(I6,6000)
WRITE(I6,6100)
IF(ITYPEO.EQ.4)GO TO 400
IF(CH2.EQ.BLANK)GO TO 300
WRITE(I6,6200)NCASE,ITYPEO,THICKX,FLGRAD,THICKY,CRHOX,CHX,CRHOY,CH
*Y
GO TO 500
DATA BLANK/6H      /
300 CONTINUE
WRITE(I6,7600)NCASE,ITYPEO,THICKX,FLGRAD,THICKY,CRHOX,CRHOY,CHY
GO TO 500
400 CONTINUE
WRITE(I6,7700)NCASE,ITYPEO,THICKX,FLGRAD,THICKY
WRITE(I6,7300)
WRITE(I6,7400)FLRHOX,FRRHOY,FLROXY,C11R,C22R,C12R,C21R
WRITE(I6,6300)
WRITE(I6,6400)STRING,RINGL,BETAF
RETURN
500 WRITE(I6,6300)
WRITE(I6,6400)STRING,RINGL,BETAF
RETURN
5000 FORMAT(1H1,31X,62HGENERAL INSTABILITY OF ECCENTRICALLY STIFFENED C
*YLINDERS 4267)
5100 FORMAT(1H0,32X,10A6)
5200 FORMAT(1H0,5HINPUT,4X,6HNUMBER,3X,8HPOISSONS,13X,5HLOWER,8X,5HUPPE
*R,7X,5HLOWER,7X,5HUPPER,13X,9HETA SUB P,4X,3HC12,5X,3HC21,5X,3HA12
*)
5300 FORMAT(1X,6HOPTION,2X,8HOF CASES,3X,5HRATIO,5X,3HE/G,4X,11HC SUB A
*LPHA,2X,11HC SUB ALPHA,2X,10HC SUB BETA,2X,10HC SUB BETA,2X,6HNRH
*OY,3X,6HOPTION,4X,6HOPTION,2X,6HOPTION,2X,6HOPTION)
5400 FORMAT(1X,A5,5X,14,5X,F5.3,2X,F7.3,5X,F6.3,7X,F6.3,6X,F6.3
*,6X,12,7X,I1,9X,I1,7X,I1,7X,I1)
5500 FORMAT(1H0,22X,12HMAX ORDINATE,10X,9HSCREENING,15X,11HMIN NO CIRC)
5600 FORMAT(25X,9HFOR PLOTS,12X,7HCUT-OFF,16X,10HHALF-WAVES,11X,7HN SUB
*R)
5700 FORMAT(1H0,22X,1PE10.3,11X,1PE12.5,15X,0PF5.2,16X,I2)
5800 FORMAT(1H0,42X,40HR/RHO Y RATIOS FOR AUTOMATIC SEQUENCING)
5900 FORMAT(1H0,(8(1PE12.5,4X),/1X))
6000 FORMAT(1H0,2X,4HCASE,9X,6HOUTPUT)
6100 FORMAT(2X,6HNUMBER,8X,6HOPTION,7X,9HT BAR X/T,8X,3HL/R,8X,9HT BAR
*Y/T,5X,11HC SUB RHO X,4X,8HC SUB HX,5X,11HC SUB RHO Y,4X,8HC SUB H

```


TABLE V - Fortran Listing - Program 4267
(Continued)

```

*Y)
6200 FORMAT(1H0,2X,I4,11X,I1,8X,7(1PE12.5,2X))
6300 FORMAT(1H0,21X,17HSTRINGER LOCATION,17X,13HRING LOCATION,16X,11HBE
*TA FACTOR)
6400 FORMAT(1H0,25X,A6,A3,25X,A6,A3,17X,1PE12.5)
6500 FORMAT(1H0,58X,7HR/RHO Y)
6600 FORMAT(1H0,55X,1PE12.5)
6700 FORMAT(1H0,16X,8HCRITICAL,6X,8HCRITICAL,6X,8HCRITICAL,8X,2HLL,12X,
*2HLH,12X,2HHL,12X,2HHH,10X,6HAXISYM)
6800 FORMAT(3X,7HL/RHO X,5X,12HLOWER CASE M,2X,12HLOWER CASE N,2X,6(11H
*SIGMA BAR/E,3X))
6900 FORMAT(1H0,9(1PE12.5,2X))
7000 FORMAT(1H0,9X,8HCRITICAL,6X,8HCRITICAL,6X,8HCRITICAL,8X,2HLL,12X,2
*HLH,12X,2HHL,12X,2HHH,10X,6HAXISYM)
7100 FORMAT(8X,12HLOWER CASE M,2X,12HLOWER CASE N,2X,6(11HSIGMA BAR/E,3
*X))
7200 FORMAT(1H0,7X,8(1PE12.5,2X))
7300 FORMAT(1H0,28X,7HL/RHO X,7X,7HR/RHO Y,7X,8HL/RHO XY,6X,5HC11/R,9X,
*5HC22/R,9X,5HC12/R,9X,5HC21/R)
7400 FORMAT(1H0,27X,7(1PE12.5,2X))
7500 FORMAT(1H0,43X,1PE12.5,15X,F5.2,16X,I2)
7600 FORMAT(1H0,2X,I4,11X,I1,8X,4(1PE12.5,2X),14X,2(1PE12.5,2X))
7700 FORMAT(1H0,2X,I4,11X,I1,8X,3(1PE12.5,2X))

```

```

C
C*****
C*****
C

```

```

ENTRY PRINT2(JJ,II)
IF(ITYPE0.EQ.4) GO TO 700
IF(JJ.NE.1) GO TO 600
IF(II.NE.1)WRITE(I6,7750)
7750 FORMAT(1H1)
WRITE(I6,6500)
WRITE(I6,6600)RDRHOY(II)
WRITE(I6,6700)
WRITE(I6,6800)
600 CONTINUE
IF(SOLUT(2).NE.0.0) GO TO 650
WRITE(I6,7775)FLROX(JJ),(SOLUT(I),I=1,3),SOLUT(8)
7775 FORMAT(1H0,4(1PE12.5,2X),56X,1PE12.5)
RETURN
650 WRITE(I6,6900)FLROX(JJ),(SOLUT(I),I=1,8)
RETURN
700 WRITE(I6,7000)
WRITE(I6,7100)
IF(SOLUT(2).NE.0.0) GO TO 750
WRITE(I6,7250)(SOLUT(I),I=1,3),SOLUT(8)
7250 FORMAT(1H0,7X,3(1PE12.5,2X),56X,1PE12.5)
RETURN
750 WRITE(I6,7200)(SOLUT(I),I=1,8)
RETURN

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

C
C*****
C*****
C
      ENTRY PRINT3(KK)
      ITYPE0=0
      I5=5
      I6=6
      I8=8
      IF(KK.NE.1)GO TO 1000
      WRITE(I6,5000)
      WRITE(I6,5100)TITLE
      WRITE(I6,7800)
      WRITE(I6,7900)
      WRITE(I6,8000)RAORST,NCASES,CALPHL,CALPHU,CBETAL,CBETAU,IETAP,IC12
      *,IC21,IA12
      WRITE(I6,8100)
      WRITE(I6,8200)
      WRITE(I6,8300)SCREEN,CHWMIN,BETAF,NR
1000  WRITE(I6,8400)
      WRITE(I6,8500)
      WRITE(I6,8600)NCASE,ITYPE0,A11,A22,A12,A33,C11,C22,R
      WRITE(I6,8700)
      WRITE(I6,8800)D11,D22,D12,D33,C12,C21,L
      WRITE(I6,8900)
      WRITE(I6,9000)
      WRITE(I6,9100)(SOLUT(I),I=1,8)
      RETURN
7800  FORMAT(1H0,5HINPUT,7X,6HNUMBER,6X,5HLOWER,8X,5HUPPER,8X,5HLOWER,7X
      *,5HUPPER,7X,9HETA SUB P,4X,3HC12,9X,3HC21,9X,3HA12)
7900  FORMAT(1X,6HOPTION,5X,8HOF CASES,2X,11HC SUB ALPHA,2X,11HC SUB ALP
      *HA,2X,10HC SUB BETA,2X,10HC SUB BETA,7X,6HOPTION,4X,6HOPTION,6X,6H
      *OPTION,6X,6HOPTION)
8000  FORMAT(1H0,A5,8X,I4,7X,F6.3,7X,F6.3,7X,F6.3,6X,F6.3,10X,I1,9X,I1,1
      *1X,I1,11X,I1)
8100  FORMAT(1H0,18X,9HSCREENING,11X,11HMIN NO CIRC,19X,4HBETA)
8200  FORMAT(1X,19X,7HCUT-OFF,12X,10HHALF-WAVES,19X,6HFACTOR,11X,7HN SUB
      * R)
8300  FORMAT(1H0,16X,1PE12.5,12X,F5.2,18X,1PE12.5,11X,I2)
8400  FORMAT(1H0,1X,4HCASE,3X,6HOUTPUT)
8500  FORMAT(1X,6HNUMBER,2X,6HOPTION,6X,3HA11,11X,3HA22,11X,3HA12,11X,3H
      *A33,11X,3HC11,11X,3HC22,12X,1HR)
8600  FORMAT(1H0,1X,I4,5X,I1,5X,7(1PE12.5,2X))
8700  FORMAT(1H0,20X,3HD11,11X,3HD22,11X,3HD12,11X,3HD33,11X,3HC12,11X,3
      *HC21,12X,1HL)
8800  FORMAT(1H0,16X,7(1PE12.5,2X))
8900  FORMAT(1H0,2X,8HCRITICAL,7X,8HCRITICAL,5X,8HCRITICAL,10X,2HLL,13X,
      *2HLH,13X,2HHL,13X,2HHH,10X,6HAXISYM)
9000  FORMAT(1X,12HLOWER CASE M,3X,12HLOWER CASE N,3X,6(7HN SUB X,8X))
9100  FORMAT(1H0,1PE12.5,3X,4(1PE12.5,2X),1X,3(1PE12.5,3X))
      END

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```
SIBFTC PLOTD  M94/2,LIST
SUBROUTINE PLOT
COMMON/INTER/
```

```
*F1
*,F2
*,F3
*,F4
*,F5
*,F6
*,F7
*,F8
*,F9
*,F10
*,F11
*,F12
*,F13
*,F14
*,F15
*,SOLUT(8)
*,ARRAY(11,25)
*,FLROX(11)
*,FMMAX
```

```
C*****
```

```
COMMON/ALL/
```

```
* TITLE(10)
```

```
*,RAORST
```

```
*,NCASES
```

```
*,POISR
```

```
*,EDSIG
```

```
*,CALPHL
```

```
*,CALPHU
```

```
*,CBETAL
```

```
*,CBETAU
```

```
*,NRRHOY
```

```
*,IETAP
```

```
*,IC12
```

```
*,IC21
```

```
*,IA12
```

```
*,NR
```

```
*,IDUMP
```

```
*,ORDMAX
```

```
*,SCREEN
```

```
*,CHWMIN
```

```
,DUMP
```

```
LOGICAL DUMP
```

```
C*****
```

```
COMMON/CSTRAT/
```

```
* NCASE
```

```
*,ITYPEO
```

```
*,THICKX
```

```
*,FLGRAD
```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

*,THICKY
*,CRHOX
*,CHX
*,CRHOY
*,CHY
*,STRING(2)
*,RINGL(2)
*,FLRHOX
*,FRRHOY
*,BETAF
*,FLROXY
*,C11R
*,C22R
*,C12R
*,C21R                                ,RDRHOY(25)
EXTERNAL NXV
EXTERNAL NYV
MX=1
MY=0
CALL SMXYV(MX,MY)
L=4
CALL SETMIV(24,0,130,24)
XL=10.0
XR=1000.0
YB=0.0
YT=ORDMAX
DX=1.0
IF(ORDMAX.LT..006.AND.ORDMAX.GT..001)DY=.0001
IF(ORDMAX.GE..006)DY=.0002
IF(ORDMAX.LE..001)DY=.00002
N=1
IF(DY.EQ..0001)M=10
IF(DY.EQ..0002.OR.DY.EQ..00002)M=5
I=-N
J=-M
J=-M
NX=6
NY=6
CALL GRIDIV(L,XL,XR,YB,YT,DX,DY,N,M,1,J,NX,NY)
CALL APRNTV(0,-14,-21,21HSIGMA BAR / ELASTIC E,5,790)
CALL PRINTV(-14,14HT BAR X / T = ,133,1010)
CALL LABLV(THICKX,245,1010,-4,1,1)
CALL PRINTV(-6,6HL/R = ,551,1010)
CALL LABLV(FLGRAD,599,1010,-4,1,1)
CALL PRINTV(-14,14HT BAR Y / T = ,823,1010)
CALL LABLV(THICKY,935,1010,-4,1,1)
CALL PRINTV(-9,9HL / RHO X,554,120)
CALL RITE2V(248,77,1023,90,1,38,-1,38HCRITICAL AXIAL COMPRESSION F
*OR GENERAL,NLAST)
CALL RITE2V(239,45,1023,90,1,39,-1,39HINSTABILITY IN CYLINDERS WIT

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```

*H ECCENTRIC,NLAST)
CALL RITE2V(392,13,1023,90,1,22,-1,22HORTHOTROPIC STIFFENING,NLAST
*)
DO 1000 I=1,NRRHOY
CALL APLOTV(11,FLROX,ARRAY(1,I),1,1,1,55,IERR)
1000 CONTINUE
RETURN
END
$IBMAP ERRTR1
*
*
* USAGE- FORTRAN IV
*
*
* CALL ERRTR (ICODE)
*
*
*
* WHERE IABS(ICODE) = FORTRAN EXECUTION ERROR MONITOR CODE (FXEM).
* IF CODE .LT. 0 , THEN NO ERROR TRACE WANTED.
* IF CODE .GT. 0 , THEN ERROR TRACE WANTED.
*
* REFERENCE : COMPUTER SYSTEMS BULLETIN, NO. 90, 9 MAY 66
*
ERRTR SAVE 2,4
AXT 0,2
CLA* 3,4
STZ FLAG
TPL AGAIN
STL FLAG
SSP
AGAIN SUB =35
TXI **1,2,-1
TZE **2
TPL AGAIN
STA ALS
CLA =1
ALS ALS **
NZT FLAG
TRA OFF
ORS ERRW-1,2
RETURN ERRTR
OFF COM
ANS ERRW-1,2
RETURN ERRTR
.ERRW. CONTRL ERRW,ERRWND
ERRW PZE
PZE
PZE
ERRWND NULL

```

TABLE V - Fortran Listing - Program 4267
(Continued)

```
*      IF FLAG .EQ. 0 , THEN TURN BIT OFF.  
*      IF FLAG .NE. 0 , THEN TURN BIT ON.  
FLAG    BSS    1  
END
```

SECTION 7

REFERENCES

1. Smith, G. W. and Spier, E. E., "The Stability of Eccentrically Stiffened Circular Cylinders, Volume I - General", Contract NAS8-11181, General Dynamics Convair Division Report No. GDC-DDG-67-006, 20 June 1967.
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5. Becker, H., "Handbook of Structural Stability, Part VI - Strength of Stiffened Curved Plates and Shells", NACA Technical Note 3786, July 1958.
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7. Thielemann, W. F., "New Developments in the Nonlinear Theories of the Buckling of Thin Cylindrical Shells", Proceedings of the Durand Centennial Conference, Held at Stanford University, 5-8 August 1959, Pergamon Press, New York, 1960.
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SECTION 7

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10. Stuhlman, C. E., "Computer Program for Integrally Stiffened Cylinders Under Axial Compression", Contract NAS 8-9500, Lockheed Missiles and Space Company Report LMSC-A740914, Rev. 1, 23 October 1965.
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12. Smith, G. W. and Spier, E. E., "The Stability of Eccentrically Stiffened Circular Cylinders, Volume V - Effects of Initial Imperfections; Axial Compression and Pure Bending", Contract NAS8-11181, General Dynamics Convair Division Report No. GDC-DDG-67-006, 20 June 1967.
13. Donnell, L. H., "Stability of Thin-Walled Tubes Under Torsion", NACA Report No. 479, 1933.
14. Smith, G. W. and Spier, E. E., "Theoretical Verification and Extension of Compressive Buckling Equations for Circular Cylinders Having Eccentric Orthotropic Stiffening", Contract NAS8-11181, General Dynamics Convair Division Memo AS-D-1030, 31 January 1967.
15. Smith, G. W. and Spier, E. E., "Digital Computer Program for the Compressive Buckling Analysis of Circular Cylinders Having Eccentric Stringers and Rings", Contract NAS8-11181, General Dynamics Convair Division Memo AS-D-1035, 28 February 1967.

SECTION 7

REFERENCES
(Continued)

16. Roark, R. J., Formulas for Stress and Strain, Third Edition, McGraw-Hill Book Company, Inc., New York, 1954.
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18. Schumacher, J. G., "Development of Design Curves for the Stability of Thin Pressurized and Unpressurized Circular Cylinders", General Dynamics Convair Report No. AZS-27-275, Revision B, 22 July 1960.

APPENDIX A

BUCKLING CURVES FOR SANDWICH CYLINDERS

In order to provide a basis for the comparison of skin-stringer-ring constructions against sandwich configurations, the buckling curves of Figure 12 are presented here. These curves are based on the classical solution of Stein and Mayers [17] for axially compressed, simply supported sandwich cylinders having isotropic faces of equal thickness and an isotropic core. Since this solution employs small-deflection shell theory, the values obtained from these curves must be reduced by suitable knock-down factors to account for the influences of initial imperfections (see Volume V [12]). This practice is necessary if one is to obtain safe design values for the critical loading. The following definitions apply to the notation used in Figure 12:

$$k_{x_a} = \frac{\left(\bar{N}_x\right)_c L^2}{\pi^2 D}$$

$$Z_a = \frac{2L^2 \sqrt{(1-\nu_F^2)}}{R h}$$

$$r_a = \frac{\pi^2 D}{D_Q L^2}$$

(A-1)

$$D = \frac{E_F t_F h^2}{2(1-\nu_F^2)}$$

$$D_Q = G_c \frac{h^2}{c}$$

where,

- $(\bar{N}_x)_c$ = Critical running axial compression, lbs/in.
L = Overall length of cylinder, in.
 ν_F = Poisson's ratio of faces.
R = Mean radius of cylinder, in.
h = Distance between middle surfaces of faces, in.
 E_F = Young's modulus of faces, psi.
 t_F = Thickness of single face, in.
 G_c = Shear modulus of core, psi.
c = Core thickness, in.

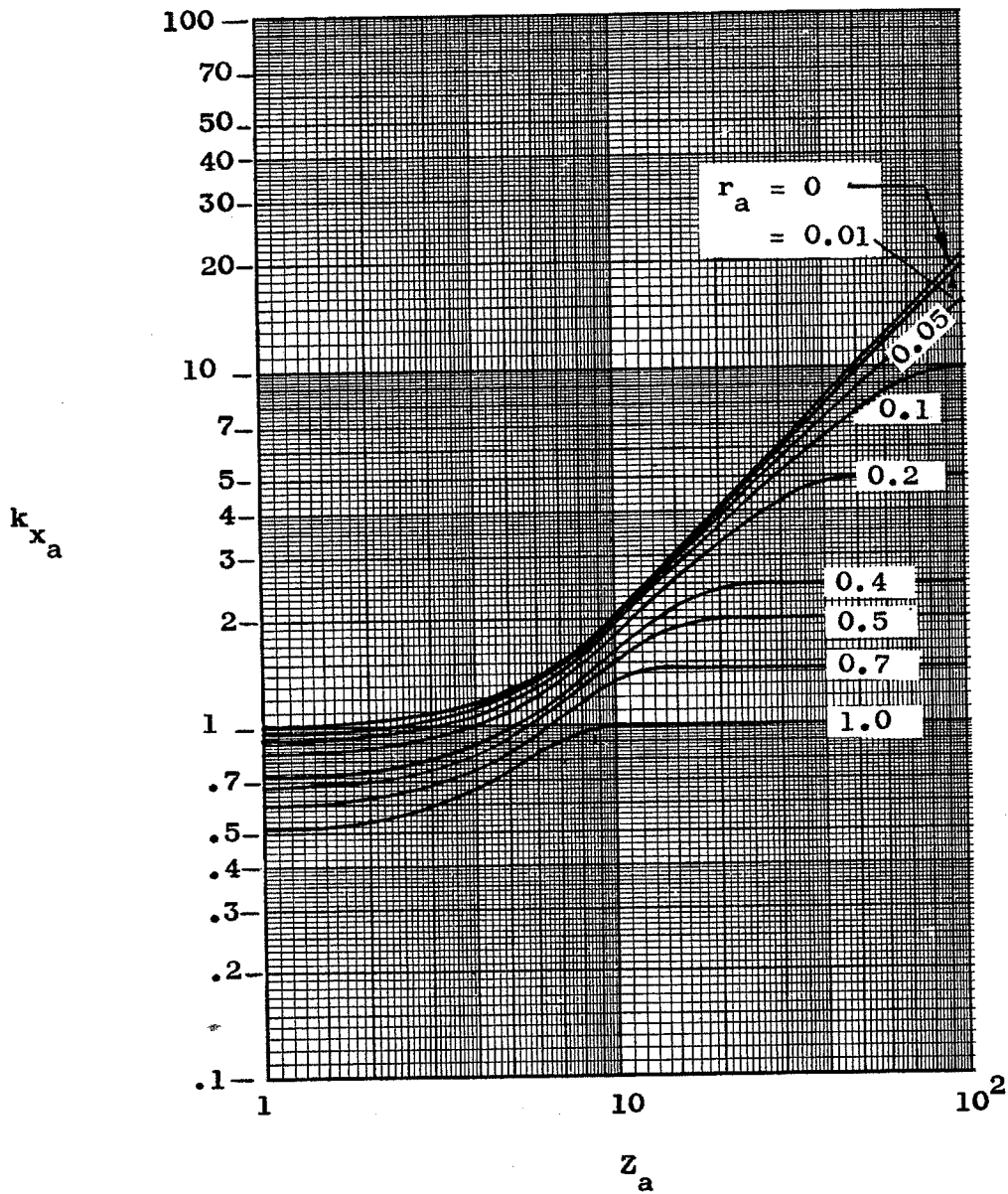


Figure 12(a) - Classical Buckling Coefficients For Simply Supported Sandwich Cylinders Under Axial Compression (Isotropic Core And Faces)

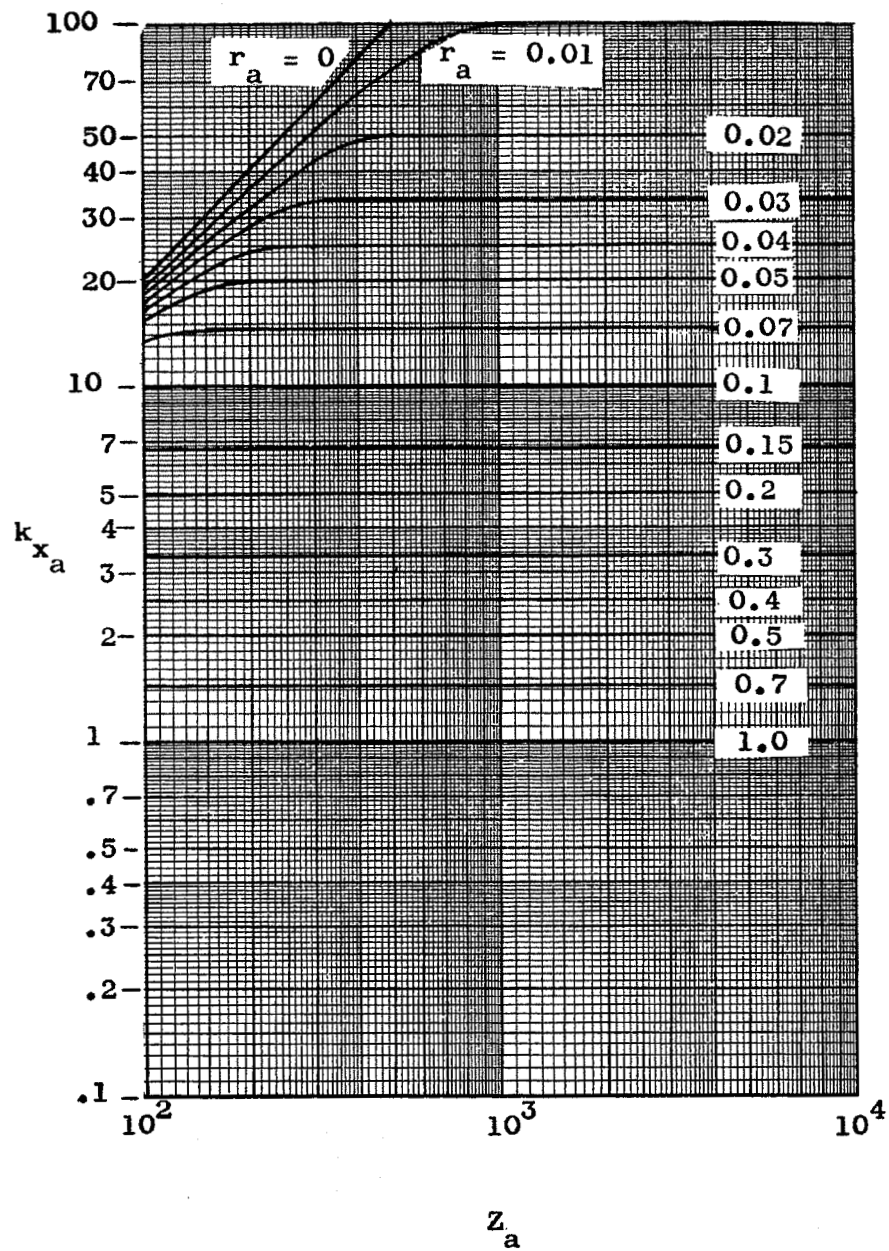


Figure 12(b) - Classical Buckling Coefficients For Simply Supported Sandwich Cylinders Under Axial Compression (Isotropic Core And Faces)

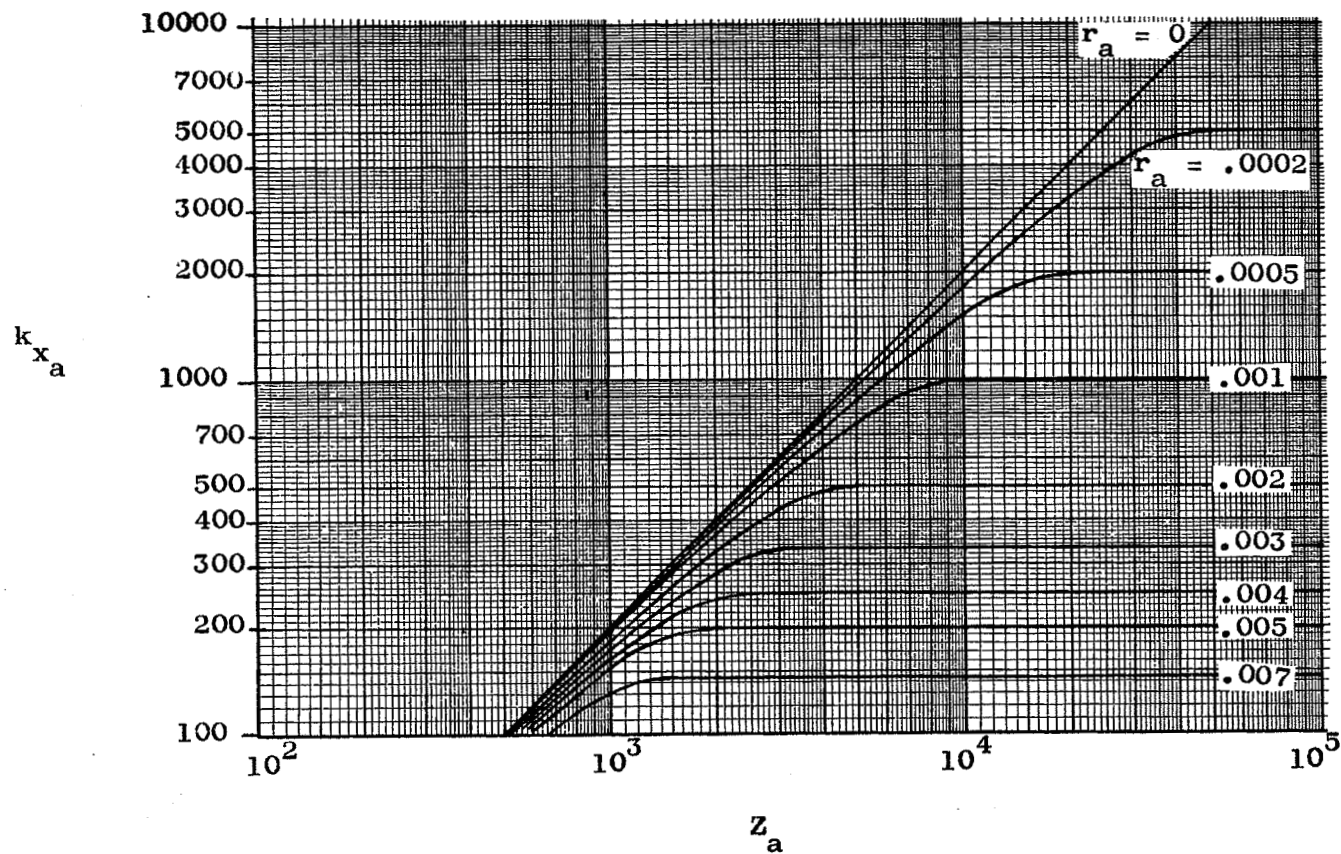


Figure 12(c) - Classical Buckling Coefficients For
Simply Supported Sandwich Cylinders
Under Axial Compression (Isotropic
Core And Faces)

APPENDIX B

BUCKLING CURVES FOR ISOTROPIC CYLINDERS

In order to provide a basis for the comparison of skin-stringer-ring constructions against isotropic cylinders, the buckling curves of Figures 13 through 18 are presented here. Each of the following loading conditions are covered:

- (a) Unpressurized under pure axial load.
- (b) Unpressurized under pure bending moment.

These curves were developed by Convair through the statistical analysis of test data [18]. Separate families are given for each of the following statistical criteria:

- (a) Best fit
- (b) 90% probability; 95% confidence
- (c) 99% probability; 95% confidence

Each family consists of separate curves for $(L/R) = 0.25, 1.0, \text{ and } 4.0$. Criterion (a) was established by means of the conventional least squares technique and gives the mean expected (50% probability) level for buckling. Half of any large array of typical test data would be expected to fall below this level. Criterion (b) may be considered to represent that there is 95% confidence that at least 90% of any large array of test points would fall on or above the related design curves. Criterion (c) may be similarly expressed for 99% probability and 95% confidence. Criteria (b) and (c) statistically correspond to the MIL-HDBK-5 "B" and "A" values, respectively.

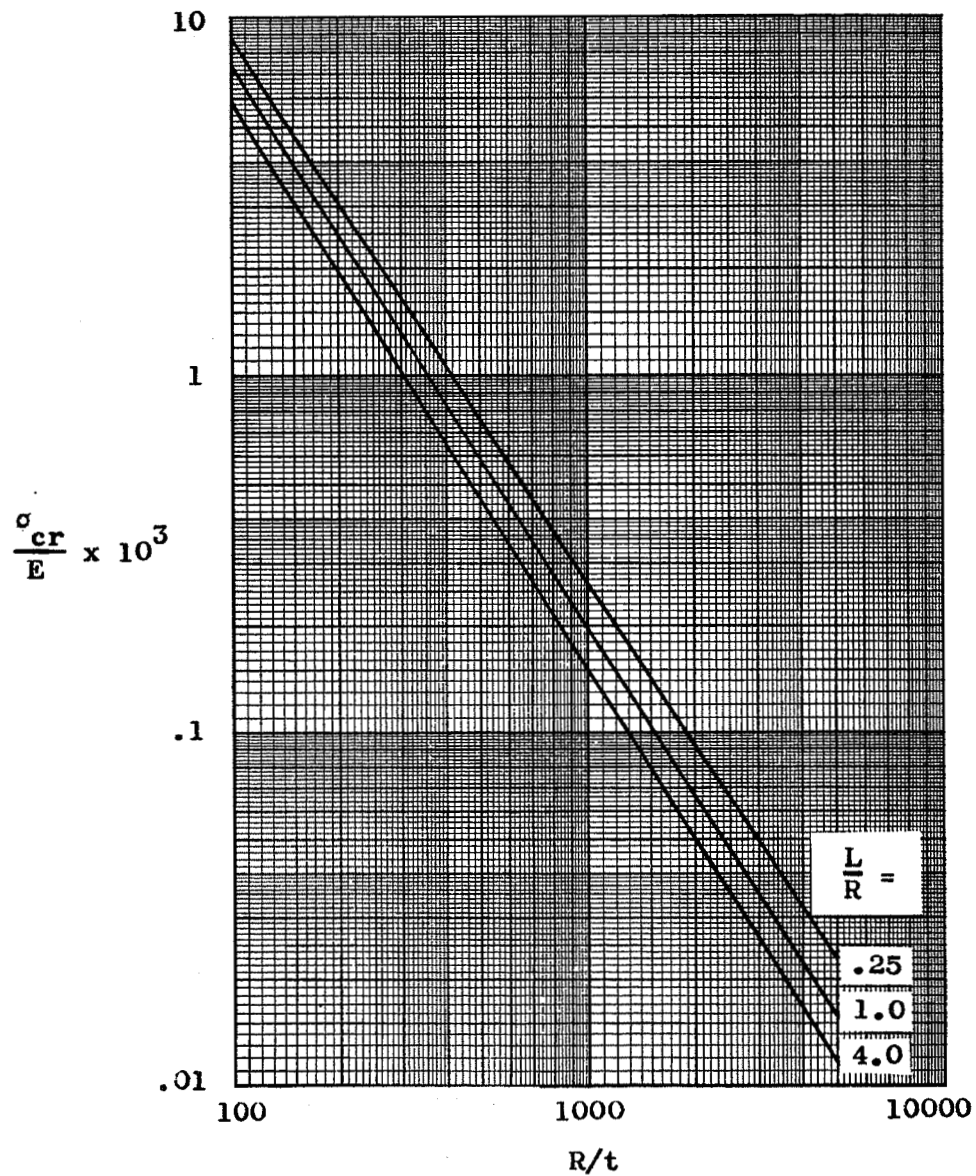


Figure 13 - σ_{cr}/E vs. R/t for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Axial Load; BEST FIT

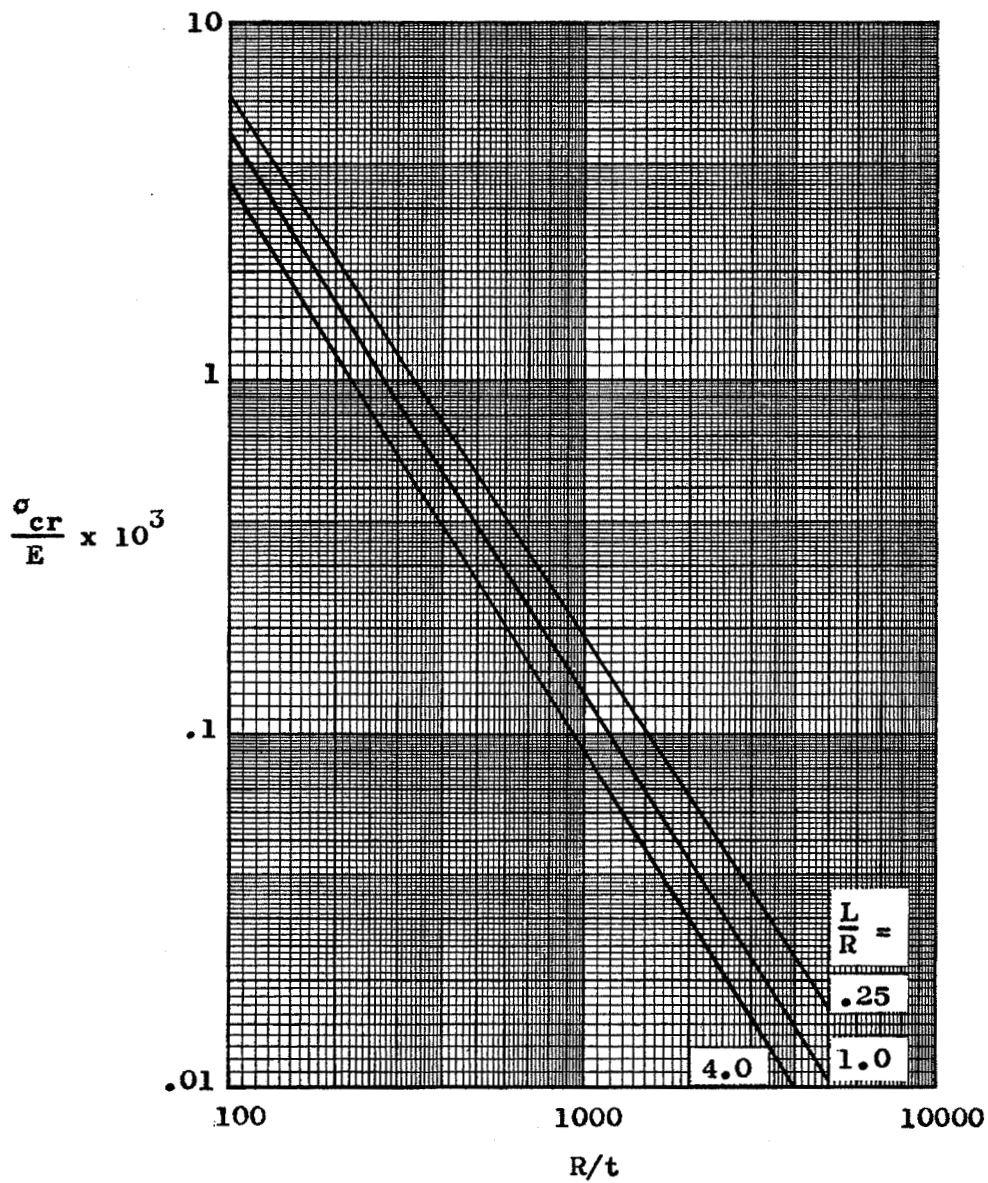


Figure 14 - σ_{cr}/E vs. R/t for Unpressurized Monocoque
Circular Cylinders (Clamped Ends) Under
Pure Axial Load; PROBABILITY = 90%
CONFIDENCE = 95%

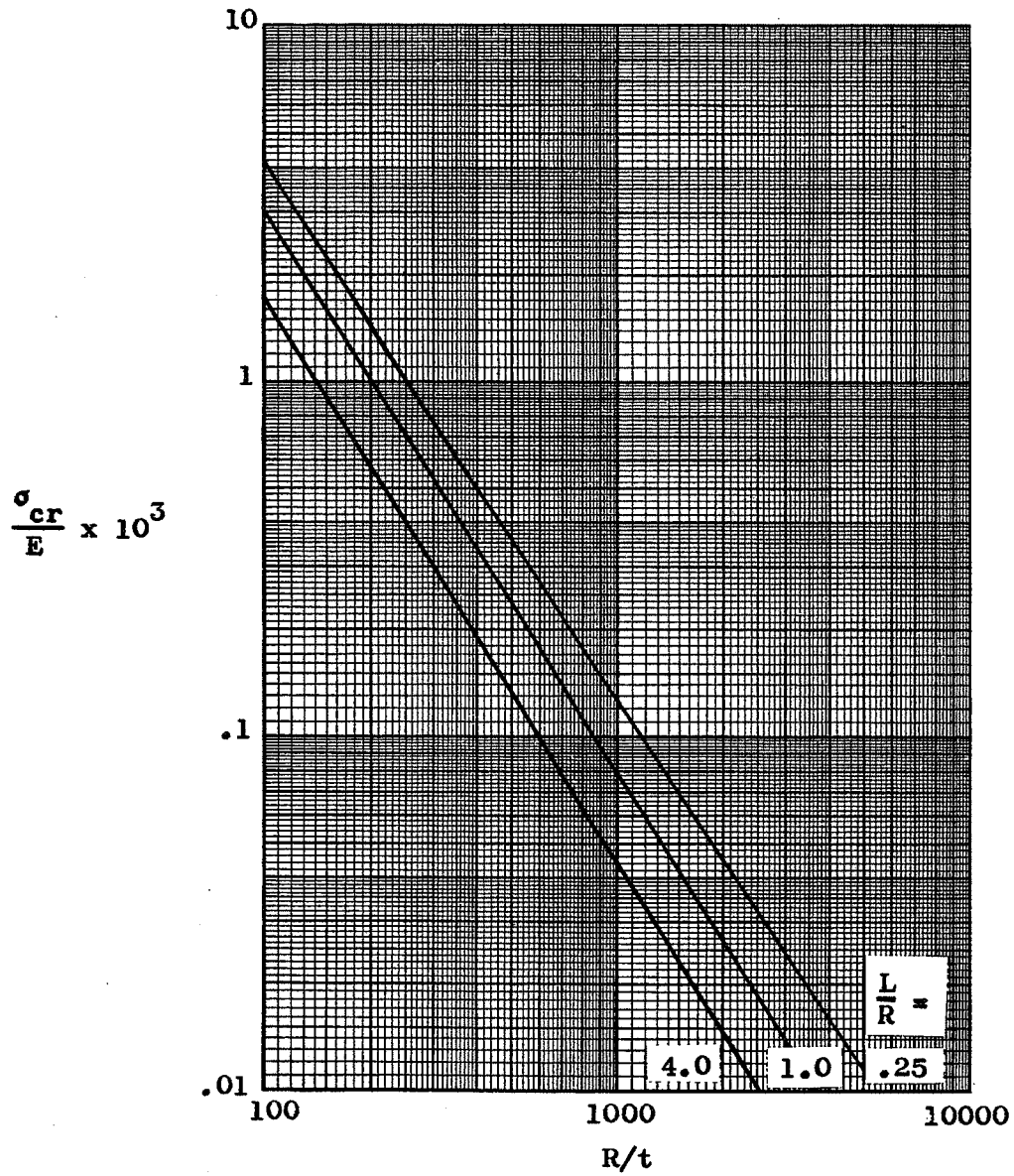


Figure 15 - σ_{cr}/E vs. R/t for Unpressurized Monocoque
Circular Cylinders (Clamped Ends) Under
Pure Axial Load; PROBABILITY = 99%
CONFIDENCE = 95%

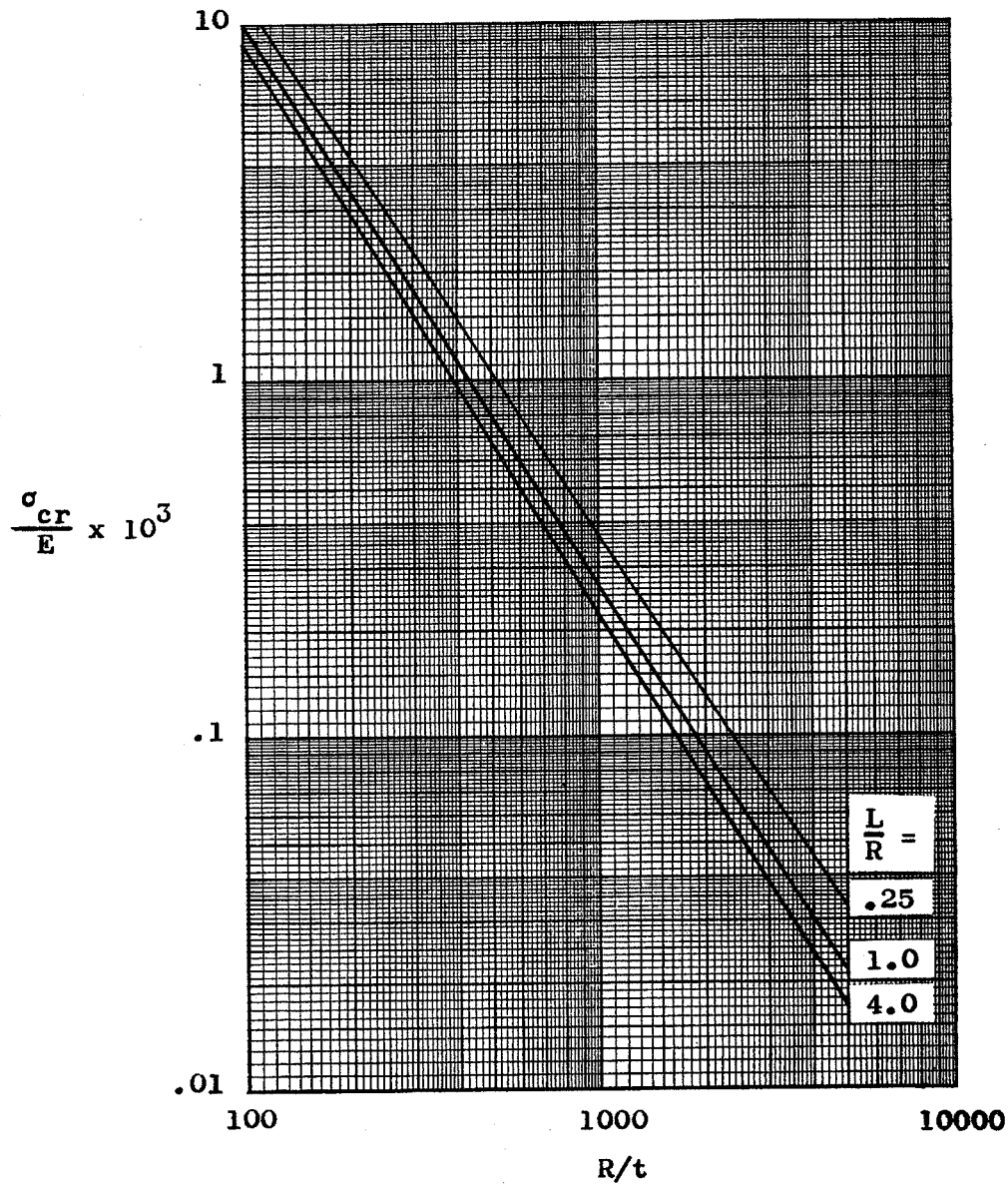


Figure 16 - σ_{cr}/E vs. R/t for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Bending Moment; BEST FIT

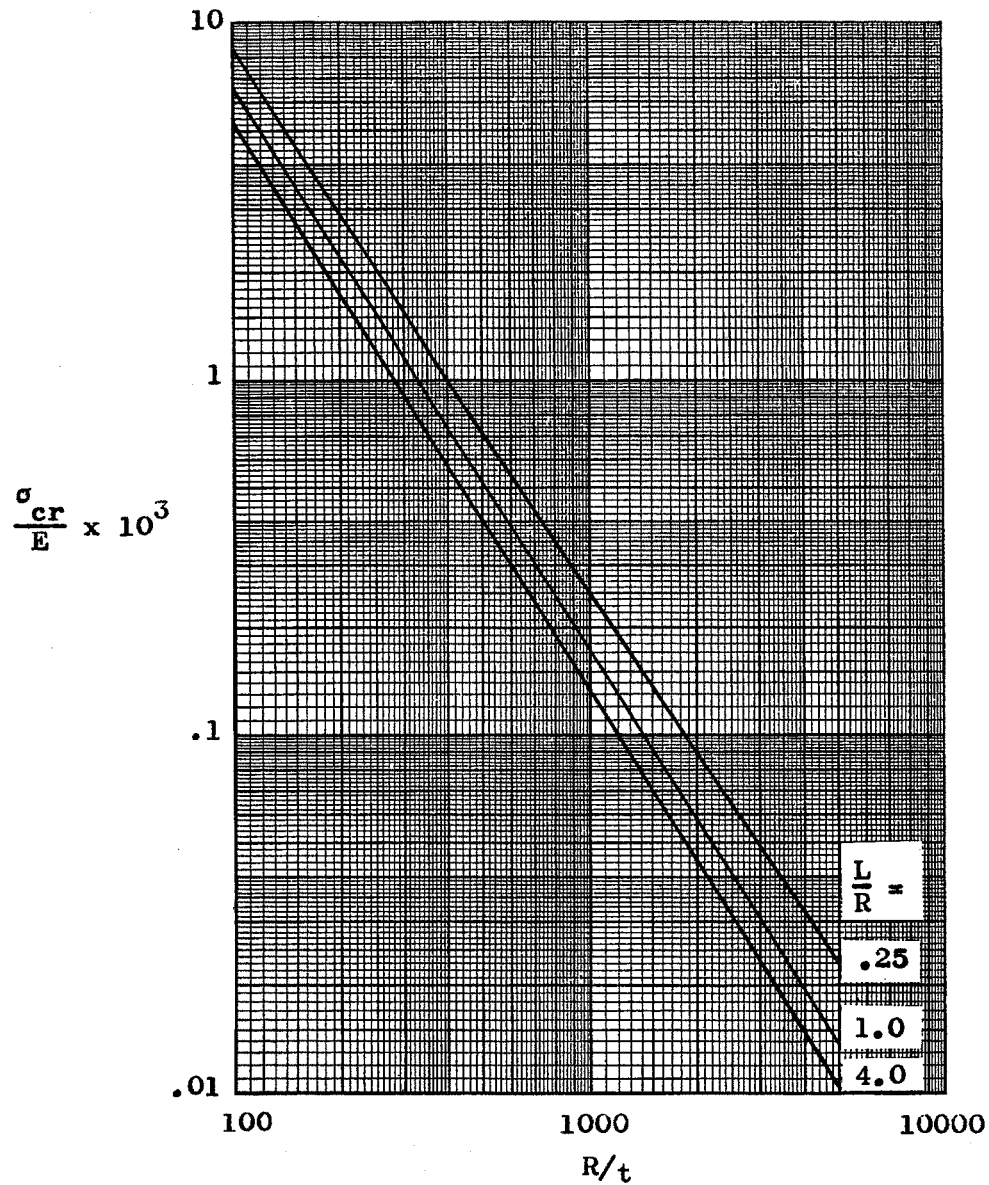


Figure 17 - σ_{cr}/E vs. R/t for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Bending Moment; PROBABILITY = 90% CONFIDENCE = 95%

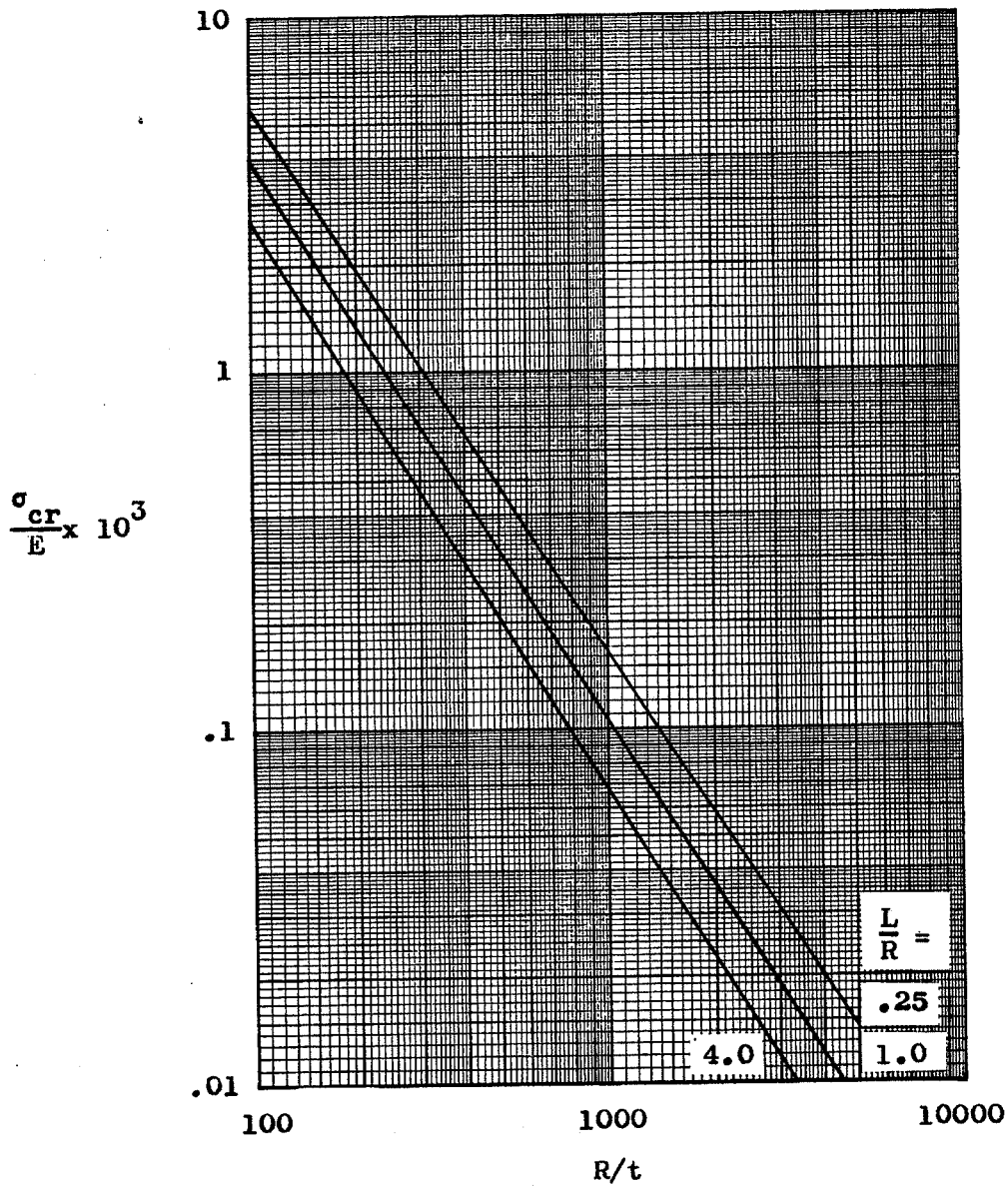


Figure 18 - σ_{cr}/E vs. R/t for Unpressurized Monocoque
Circular Cylinders (Clamped Ends) Under
Pure Bending Moment; PROBABILITY = 99%
CONFIDENCE = 95%